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OPTIMIZATION OF THE THERMAL RESISTANCE OF ROOF INSULATION IN REROOFING

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ABSTRACT

This paper presents an economic model for calculating the optimum thermal resistance for roof insulation. It applies primarily to reroofing work, because it does not account for heating and cooling equipment cost savings attributable to increased insulation.

The economic model uses current costs and heat content of fuels, heating and cooling degree days, and the installed cost of roof insulation per unit of thermal resistance. It also shows the economic penalty for deviating from the calculated optimum insulation.

Optimum thermal resistance is calculated for twenty representative cities in the United States, and is compared with ASHRAE Standard 90-75 for insulation in commercial and high rise residential buildings.

Optimum thermal resistance calculated with natural gas as the fuel is lower than called for by the ASHRAE standard; the calculated optimum thermal resistance using electricty or oil is about the same as the ASHRAE standards. The scatter of the optimum thermal resistance calculated for each city shows that the thermal resistance should be calculated using local data rather than average values.

1. INTRODUCTION

Most of the roofing systems currently in place on our public and commercial buildings were installed when roof insulation was relatively more expensive than the energy used to heat the buildings, and low first cost was the primary means of selecting the quantity of insulation installed. As a result, the quantity of insulation is frequently inadequate, causing economic loss and excessive use of our fossil fuel resources.

An obvious time for upgrading the insulation content of a roofing system is when the existing system has failed when the existing membrane and insulation are water-saturated.

The American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) has proposed a maximum thermal conductance value for roofs and ceiling in their Standard 90-75.[2] This valuable first step is the only published guide the designer of reroofing has to select the quantity of roof insulation to be used in the new system. There are some practical difficulties in using Standard 90-75, since it includes the heat conductance through the ceiling, roofing system, and associated spaces. The ceilings and structure are seldom uniform over a large building; thus, to conform to the standard, the designer must plan to use the quantity of roof insulation needed to retard the heat flow through the part of the ceiling and structure that has the maximum heat flow. The point of maximum heat flow may be difficult to locate and may not be representative of the building as a whole. Moreover, the ASHRAE Standard does not take into account the local cost of fuel or insulation.

This paper recommends an alternate method of selecting the quantity of roof insulation to be used, based on current economics and geographic exposure. The optimum insulation quantity for roofing is calculated directly with an economic model.

2. ECONOMIC MODEL

The economic model is an extension of earlier work[1]; it involves optimizing the sum of the cost of fuel needed to make up for the heat lost, the cost of the fuel needed to make up for heat gain, and the installed cost of the roof insulation. It is based on the following assumptions:

- Steady-state heat flow.
- Future annual energy cost savings equal to current cost savings—i.e., an assumed energy cost escalation rate equal to the interest, or discount rate for computing the Present Worth of future dollars.
- 20-year roof service life.
- No saving from reduced capital cost of smaller heating or cooling equipment, because this model covers existing buildings.

The model ignores radiational cooling or heating, ventilation rates, lighting load, and wall losses, since these are all characteristic of the existing building and will be unchanged by the reroofing work.

Heating fuel cost (in dollars per square foot of roof area for a period of 20 years) is proportional to the fuel cost and the climate (expressed in heating degree-days), and inversely proportional to the thermal efficiency of the heating system, the heating value of the fuel, and the thermal resistance. This results in the formula:

(1) $P_f = 480\ M \bullet H / e \bullet B \bullet R$ ($\$/ft^2 \bullet 20$ years)

where:

P_f = the heating fuel cost required to make up for the heat lost ($\$/ft^2 \bullet 20$ years)
M = the market cost of fuel in conventional units ($/unit)
B = the heating value contained in one conventional unit (Btu/unit)
e = the efficiency of the heat plant and its distribution system (per cent, as a decimal)
H = the annual degree days typical for the building's location (degree days)
R = the thermal resistance of the insulation ($h \bullet ft^2\ °F/Btu$)

The factor of 480 is the conversion from hours to 20 years (24 x 20), to convert degree-days to the average temperature difference between 65°F (18°C) and the ambient exterior temperature.

Cooling cost (in dollars per square foot of roof area and for 20 years) is calculated similarly to the heating cost. This results in the formula:

(2) $P_c = 480\ E \bullet C / e_c \bullet B_c \bullet R$

where:

P_c = the cooling fuel cost required to make up for the heat gained ($\$/ft^2 \bullet 20$ years)
E = the market cost of cooling fuel in conventional unit ($/unit)
B_c = the Btu content of one conventional unit (Btu/unit)
e_c = the efficiency of the cooling plant and its distribution system (as a fraction)
C = the annual cooling degree days typical for the building's location (degree days)

On the basis of our cost figures, we estimate the installed insulation cost P is shown by the formula:

(3) $P_i = J \bullet R$, ($\$/ft^2 \bullet 20$ years)

where:

P_i = the cost of roof insulation installed ($\$/ft^2 \bullet 20$ years)
J = $0.057 to 0.083 ($/ft • 20 years • R); $0.075 mean was used in our calculations*

Type of Insulation	Perlite/Organic	Glass Fiber	Urethane
Conductivity k (Btu-in./h ft^2 °F)	0.36	0.25	0.15
For thickness of 1" ($\$/ft^2$ R)	0.083	0.079	0.064
For thickness of 1" ($\$/ft^2$ R)	0.075	0.071	0.057

The sum of these costs is the 20-year cost of the fuels and installed insulation:

$$(4)\quad P = P_f + P_c + P_i = \frac{480\ M \bullet H}{e \bullet B \bullet R} + \frac{480\ E \bullet C}{e_c \bullet B \bullet R} + J \bullet R$$

By differentiating this equation in respect to R and setting the differential equal to zero, the equation for the optimum thermal resistance (R_o) is :

$$(5)\quad R_o = \sqrt{\frac{480\ M \bullet H}{e \bullet B \bullet J} + \frac{480\ E \bullet C}{e_c \bullet B_c \bullet J}} = \sqrt{(R_h)^2 + (R_c)^2}$$

where:

R_h = the optimum thermal resistance based on heating
R_c = the thermal resistance based on cooling
This formula is used throughout this paper.

3. ECONOMIC PENALTIES

The economic penalty for deviating from optimum thermal resistance, shown by the model, expressed as the percentage increase of the optimum cost is:

$$(6)\quad p = 50n + \frac{50}{n} - 100$$

where p is the percent increase in cost (the penalty at optimum cost = 0) for a n fraction of the optimum thermal resistance ($n = R_{actual} / R_{optimum}$). This relationship is shown in the graph on Figure 1; it is developed from equation 4 as the change in cost (P) x 100 divided by the optimum cost.

*Any other cost figure can be substituted as appropriate for geographic location and date; our cost range was derived from the following prices, all given in $\$/ft^2 \bullet R$, i.e., based on cost per square foot for the insulation thickness which produces R = 1 ($h \bullet ft^2 \bullet °F/Btu$).

The shape of the curve in Figure 1 shows that there is a lower penalty for using too much insulation than there is for using too little. As an example, assume optimum thermal resistance for a given roof at 11.1 (equivalent to 4 in. of fiberboard roof insulation). For 5 in. of fiberboard the penalty is 2.5% vs. a penalty of 4.2% for 3 in. of fiberboard. As deviation from the optimum increases, the penalty increases exponentially. Assume again a roof that requires 4 in. thick fiberboard roof insulation, with only 2 in. The economic penalty is 25% of the 20-year cost of the optimum insulation thickness.

4. GEOGRAPHIC DATA

Data were obtained for twenty cities in the United States[3] spanning 0 to 10,000 heating degree-days, as shown in Figure 2. When the heating degree days are plotted vs. the log of the cooling-degree days (Figure 3), a linear relationship is approximated that has a coefficient of correlation of slightly better than 0.99 (1.0 is a "perfect" coefficient of correlation). Determined by the least squares methods, this empirical equation is:

(7) $\log C = 3.70736 - .00015515 * H$; or:

(8) $C = 5098/1.00036^{H}$

This relationship permits us to calculate the "ideal" thermal resistances for 0 to 10,000 heating degree days.

Figure 4 shows separate curves for the thermal resistance required at 0 to 10,000 heating degree days for electric, oil, and gas heating fuel; it shows the thermal resistance required for electric cooling and optimum thermal resistance (for both cooling and heating) for each heating fuel with electric cooling.

The following average values were assumed for all locations:

Gas—$0.15/100,000 Btu (Therm), 70% efficiency
Oil—$0.40/140,000 Btu. 60% efficiency
Electricity—$0.0179/3413 Btu, 100% efficiency (1 kWh = 3413 Btu)
Insulation cost—$0.075/R installed

As might be expected, more insulation is required to obtain the optimum thermal resistance, when electricity is used for both heating and cooling, than when oil or gas are used as the heating fuel.

The fuel costs are estimates based on ASHRAE data; while adequate for these examples, actual local current costs should always be used.

To test the scatter of the data, the optimum thermal resistance at twenty cities was calculated using the actual heating and cooling degree days[3] (rounded to the nearest 10 degree days), gas for heating, and electric cooling. The cost of gas and electricity at each location is shown in Figure 5.

The optimum thermal resistance calculated for each location is shown in Figure 6; these are shown with the average "ideal" thermal resistance curve calculated earlier.

The wide scatter of the data shows the danger of using an averaging curve rather than individual calculation for each location. Part of the scatter is caused by the variation in fuel costs (gas is $0.07 to $0.26 per Therm; electricity is $0.0075 to $0.0263 per kilowatt hour); part of the scatter is due to the significant departure of the actual cooling degree days at Los Angeles, CA, San Francisco, CA, and Seattle, WA, from the empirical relationship used in the "ideal" case shown in the curves.

5. COMPARISON WITH ASHRAE STANDARD 90-75

ASHRAE Standard 90-75 lists a range of maximum thermal conductance for highrise residential and commercial buildings based on the local heating degree-days. The maximum thermal conductance values in the standard include thermal resistance values for ceilings, enclosed air spaces, roof insulation, and the inner and outer air films. As such, it is not directly comparable with the data from our model that includes only values for the roof insulation.

While not directly comparable, it follows that the thermal resistance values in the ASHRAE Standard should be substantially higher than the optimum. The ASHRAE Standard values (converted to minimum thermal resistance) are shown in Figure 7 together with the average optimum thermal resistance values obtained from the economic model.

Figure 7 shows that the ASHRAE Standard is at approximately the same level as the values calculated with the model. In terms of economic penalty, the average difference between these curves is less than 2 percent. The maximum penalty is 8% at 8000 heating degree days between the ASHRAE Standard and the calculated mean optimum thermal resistance calculated for gas heat.

6. CONCLUSIONS

Despite the very conservative assumptions used in the model for this report, the model recommends more insulation than implied by ASHRAE Standard 90-75.

The model reported is simple to use; the necessary variables can be readily obtained from local sources, from the Environmental Data Service, and from the ASHRAE Fundamentals Handbook[4]. If local cost data for fuel is unavailable, the ASHRAE minima should be used for roof insulation alone, because over-insulating has a lower economic penalty than underinsulating.

In general, average curves are useful in understanding general principles; but, the scatter of local data caused by fuel cost variation and less than accurate empirical relationships, make individual calculations advisable for each building.

REFERENCES

(1) Yamamoto, G.S. (Ed). *"Energy Conservation in New Building Design,"* Standard 90-75, American Society of Heating, Refrigerating, and Air Conditioning Engineers, New York, 1975.

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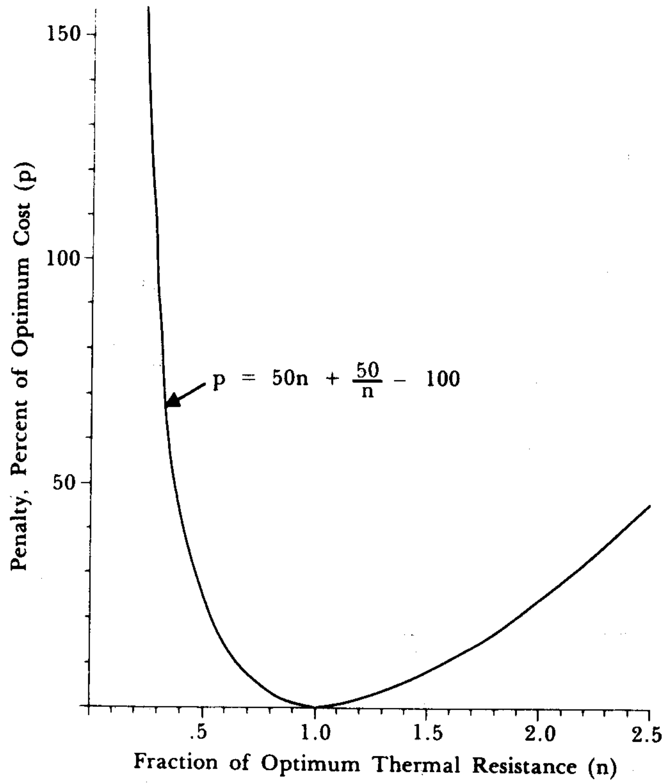

FIGURE 1 - PENALTY, % OF OPTIMUM COST VS. FRACTION OF OPTIMUM THERMAL RESISTANCE

Location	ANNUAL DEGREE DAYS Heating	ANNUAL DEGREE DAYS Cooling	Location	ANNUAL DEGREE DAYS Heating	ANNUAL DEGREE DAYS Cooling
Charlotte Amalie	0	5427	Elizabeth, NJ	5017	953
Miami, FL	206	4038	Bridgeport, CT	5461	735
Orlando, FL	704	3447	Port Jervis, NY	6087	622
Savannah, GA	1952	2317	Amherst, MA	6576	511
Augusta, GA	2547	1995	Binghamton, NY	7285	369
Atlanta, GA	3095	1598	Hanover, NH	7680	327
Williamsburg, VA	3671	1345	Massena, NY	8237	343
Richmond, VA	3939	1353	Old Town, ME	8648	209
Washington, DC	4211	1415	Presque Isle, ME	9135	168
Baltimore, MD	4729	1108	Caribou, ME	9632	128

FIGURE 2

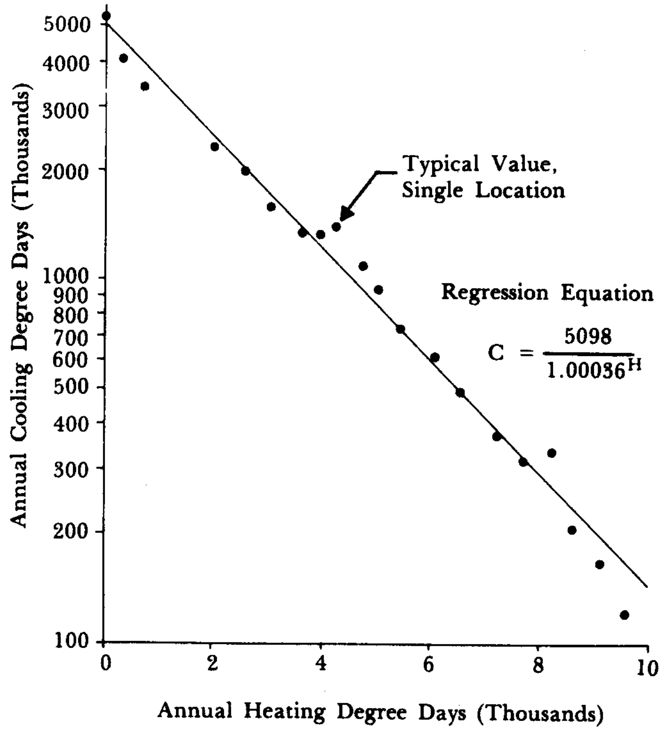

FIGURE 3–ANNUAL COOLING DEGREE DAYS VS. ANNUAL HEATING DEGREE DAYS

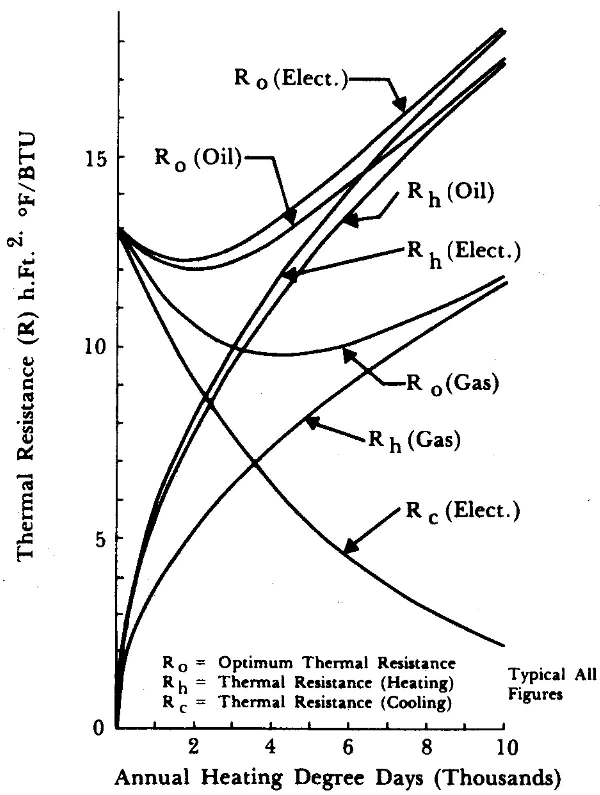

FIGURE 4–THERMAL RESISTANCE VS. ANNUAL HEATING DEGREE DAYS

Assumed: heating efficiency = 0.70 (70%)
cooling efficiency = 1.00 (100%)
insulation cost = $0.075/R

FIGURE 5

Location	Heating Degree Days	Gas $/Therm	Cooling Degree Days	Electricity $/kWh	Optimum Thermal Resistance
Atlanta, GA	3100	0.1200	1590	0.0145	8.8
Billings, MT	7270	0.1103	500	0.0140	9.3
Boston, MA	5620	0.2398	660	0.0260	12.5
Buffalo, NY	6930	0.1548	440	0.0155	10.5
Chicago, IL	6130	0.1677	930	0.0199	11.3
Cleveland, OH	6150	0.1280	610	0.0200	9.7
Denver, CO	5510	0.0740	740	0.0198	8.0
Detroit, MI	6240	0.1224	650	0.0169	9.6
Houston, TX	1430	0.1286	2890	0.0148	9.8
Kansas City, MO	5160	0.0901	1420	0.0222	10.1
Los Angeles, CA	1820	0.1057	620	0.0129	5.7
Miami, FL	210	0.2821	4040	0.0154	11.0
Minneapolis, MN	8310	0.1380	530	0.0175	11.1
New York, NY	4850	0.1845	1070	0.0263	11.6
Philadelphia, PA	4870	0.1932	1100	0.0232	11.6
Phoenix, AZ	1550	0.1231	3510	0.0196	12.1
Portland, ME	7500	0.2625	250	0.0172	13.7
St. Louis, MO	5440	0.1241	1640	0.0214	11.3
San Francisco, CA	3040	0.0908	110	0.0128	5.3
Seattle, WA	5190	0.1659	130	0.0075	9.0

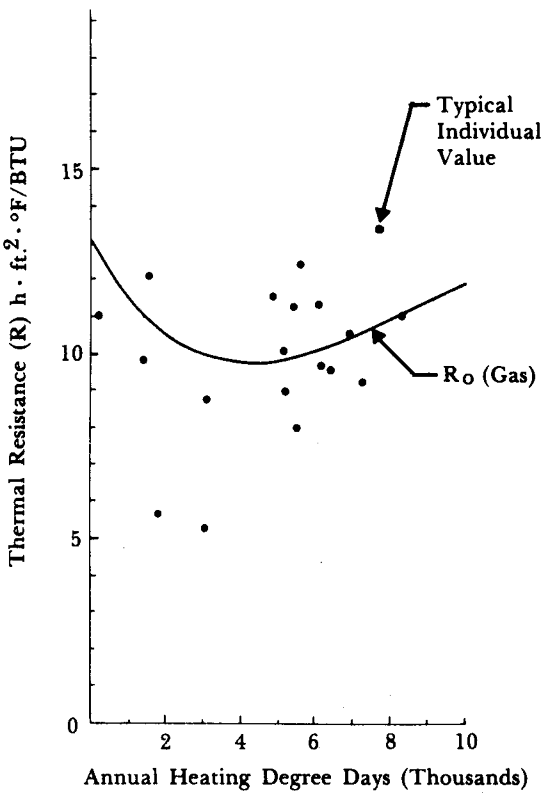

FIGURE 6 - OPTIMUM THERMAL RESISTANCE VS. ANNUAL HEATING DEGREE DAYS

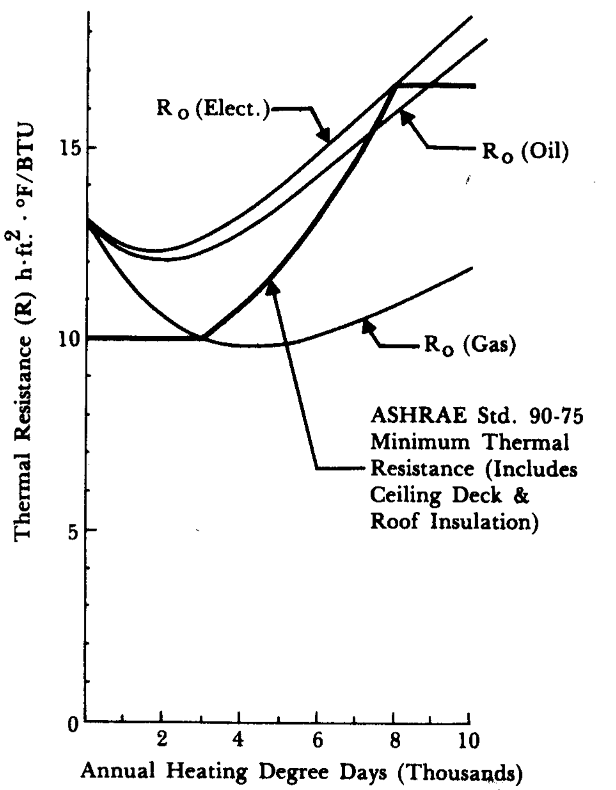

FIGURE 7 - THERMAL RESISTANCE VS. ANNUAL HEATING DEGREE DAYS