RESTRAINED AND NON-RESTRAINED DEFORMATION OF VISCO-ELASTIC BODIES

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DISBONDING LAYER EFFECT

Joint sealing is one of the waterproof methods generally used in China in such structures as roofs, basements, air-raid shelters, dams, pools, bridges, etc. As characterized by their viscoelasticity which endows material with ability to sustain deformation, asphalt, rubber, polychloroprene and similar materials usually are applied as sealing compounds *(Figure 2)*.

In Figure 1, the cement mortar infill is applied to control the thickness of the sealing material.

A common practice is to apply the sealing material directly on the cement mortar infill. Thus, restrained deformation results from the opening of the joint *(Figure 2)*. In this case, the "free length"[1,2] of the sealing material is very small, so the permissible deformation is relatively low. Furthermore, the amount of deformation is not related to the joint width.

To increase the deformability of the sealing materials, a disbonding layer such as wax paper, oil paper, or sand layer, is employed in China. In this way deformation is non-restrained by a "free length" of the sealing material extending the entire width of the joint, and its deformability is greatly increased *(Figure 3)*.

In order to establish the deformation pattern, a quantitative analysis is made of the stress and strain relating to these two cases of restraint.

RHEOLOGICAL PROPERTIES OF SEALING MATERIAL

Data obtained through measurement

Sample: tar, polychloroprene

Equipment: auto-control static rheological meters

Size: $1.85 \times 4.65 mm^2$

Sampling frequency: 3 seconds

During experiments, computers controlled data sampling and printed out a curve as shown in Figure 4.

Column 1: Time

Column 2: Initial data (dimensionless shear strains)

Model and Parameters

From the curve in Figure 4, we come to a preliminary judgement that this material can be presented by Model (H-N)-(H/N)-(H/N). *(Figure 5.)*

With the help of computers, programming the physical parameters of the said rheological model, we get,

$E_1 = 952.380953\ g/cm^2$, $E_3 = 120.275236\ g/cm^2$,
$E_2 = 74.2011482\ g/cm^2$, $\eta_1 = 1875.67373\ g.s/cm^2$,
$\eta_2 = 179.25547\ g.s/cm^2$,
$\eta_3 = 1931.73595\ g.s/cm^2$,

They correspond to P1, P2, P3, P4, P5, and P6 in Figure 4.

Rheological and creep equations

From Figure 5, we have[3],

$$\dot{\epsilon}_1 = \frac{\dot{\sigma}}{E_1} + \frac{\sigma}{\eta_1}, \quad \sigma = E_2\epsilon_2 + \eta_2\dot{\epsilon}_2$$

$$\sigma = E_3\epsilon_3 + \eta_3\dot{\epsilon}_3, \quad \epsilon = \epsilon_1 + \epsilon_2 + \epsilon_3.$$

(ϵ_1, ϵ_2 and ϵ_3 represent respectively the deformation of Maxwell element and first and second Kelvin elements).

By Laplace transformation, we easily find,

$$\bar{\epsilon} = \left(\frac{1}{E_1} + \frac{1}{\eta_1 s} + \frac{1}{E_2 + \eta_2 s} + \frac{1}{E_3 + \eta_3 s} \right) \bar{\sigma} \quad (1)$$

$\bar{\epsilon}$, $\bar{\sigma}$ is the Laplace transform of ϵ and σ, that is, the phase function.

Through inverse computation, we get the rheological equation:

$$\alpha_3 \frac{d^3\sigma}{dt^3} + \alpha_2 \frac{d^2\sigma}{dt^2} + \alpha_1 \frac{d\sigma}{dt} + \alpha_0\sigma = \beta_3 \frac{d^3\epsilon}{dt^3} + \beta_2 \frac{d^2\epsilon}{dt^2} + \beta_1 \frac{d\epsilon}{dt}$$

It can be simplified as

$$(\alpha_3 D^3 + \alpha_2 D^2 + \alpha_1 D + \alpha_0)\sigma = (\beta_3 D^3 + \beta_2 D^2 + \beta_1 D)\epsilon \quad (2)$$

where the operator $D^n = \frac{d^n}{dt^n}$

$\alpha_3 = \eta_1\eta_2\eta_3$,
$\alpha_2 = E_1\eta_1\eta_2 + E_1\eta_2\eta_3 + E_1\eta_3\eta_1 + E_2\eta_1\eta_3 + E_3\eta_1\eta_2$,
$\alpha_1 = \eta_1 E_1 E_2 + \eta_1 E_2 E_3 + \eta_1 E_3 E_1 + \eta_2 E_1 E_3 + \eta_3 E_1 E_2$,
$\alpha_0 = E_1 E_2 E_3$,
$\beta_3 = E_1\eta_1\eta_2\eta_3$,
$\beta_2 = \eta_1\eta_2 E_1 E_3 + \eta_1\eta_3 E_1 E_2$,
$\beta_1 = \eta_1 E_1 E_2 E_3$

When σ equals $\sigma_{\cdot}$ (constant), put equation (1) through inverse Laplace transformation. We get the creep equation:

$$\epsilon = \sigma_0 \left[\frac{1}{E_1} + \frac{t}{\eta_1} + \frac{1}{E_2}(1 - e^{-\frac{E_2}{\eta_2}t}) + \frac{1}{E_3}(1 - e^{-\frac{E_3}{\eta_3}t}) \right]$$

With a computer to build the model automatically, first we get a print-out, and plot a curve *(Figure 4)*. Comparing it with the creep curve, we find they coincide quite well. This verifies that our preliminary judgment that the Chinese viscoelastic mastic can be referred to as an (H-N)-(H/N)-(H/N) body.

DEFORMATION PATTERN OF SEALING MATERIAL IN NON-RESTRAINED CONDITION

For a disbonded seam-filling mastic using wax paper, oil paper or sand layer, the cement mortar infill breaks, when the joint opens. But, due to the disbonding layer, the sealing material is subjected to a uniform tensile stress $\sigma_0(t)$ in a non-restrained condition. This stress remains unchanged hereafter *(Figure 6)*.

In the coordinate system of Figure 6, $\sigma\chi=\sigma_0(t)$, $\sigma_z=0$, as the "slab" is relatively wide, we assume $\epsilon_y 0$, (y-axis is perpendicular to the paper surface), then $\sigma_y=\mu\sigma\chi$ (μ is Poisson's ratio) the stress and strain in the seal material can be expressed simply in terms of a tensor:

Tensor of stress $\sigma_{ij}=\alpha_m\delta_{ij}+S_{ij}$ (4)

Tensor of strain $\epsilon_{ij}=\epsilon_m\delta ij+e_{ij}$ (5)

Where average stress $\sigma_m=\dfrac{\sigma_x+\sigma_y+\sigma_z}{3}$

average strain $\epsilon_m=\dfrac{\epsilon_x+\epsilon_y+\epsilon_z}{3}$

$$\delta_{ij}=\begin{cases}1, i=j\\ 0, i\neq j\end{cases}$$

Introducing the constitutive relation of viscous and elastic bodies[4], we have:

$$\left.\begin{array}{l}S_{ij}=2Ge_{ij}\\ \sigma_m=3K\epsilon_m\end{array}\right\}$$

$$\left.\begin{array}{l}S_{ij}=2\eta e_{ij}\\ \sigma_m=3K\epsilon_m\end{array}\right\}$$

where volume modulus

$$K=\frac{E}{3(1-\mu)}$$

Extending (2) to space, we have,

$$(\propto_3D^3+\propto_2D^2+\propto_1D+\propto_0)S_{ij}=2(B_3D^3+B_2D^2+B_1D)e_{ij} \quad (6)$$

When i=j, using (4) and (5), we get,

$$\left.\begin{array}{l}(\propto_3D^3+\propto_2D^2+\propto_1D+\propto_0)(\sigma_x-\sigma_m)=2(B_3D^3+B_2D^2+BD)(\epsilon_x-\epsilon_m)\\ D\sigma_m=3KD\epsilon_m\end{array}\right\} \quad (7)$$

Put this through the Laplace transformation, considering the known condition of stress and strain, we get:

$$2(B_3S^3+B_2S^2+B_1S)\left(\frac{2\bar{\epsilon}_x-\bar{\epsilon}_j}{3}\right)$$

$$=\frac{2-\mu}{3}(\propto_3S^3+\propto_2S^2+\propto_1S+\propto_0)\frac{\sigma_0}{3}$$

$$K(\bar{\epsilon}_x+\bar{\epsilon}_z)=\frac{1+\mu}{3S}\sigma_o$$

Solve the latter equation for $\bar{\epsilon}_z$. Substitute it into the former equation. Considering (2), we find:

$$\bar{\epsilon}_x=\left\{\frac{1+\mu}{9K}\cdot\frac{1}{S}+\frac{2\times\mu}{6}\left[\frac{1}{E_1S}+\frac{1}{\eta_1}\frac{1}{S^2}+\frac{1}{S(E_2+\eta_2S)}+\frac{1}{S(E_3+\eta_3S)}\right]\right\}\sigma_0$$

By inverse Laplace transformation, it yields:

$$\bar{\epsilon}_x=\left\{\frac{1+\mu}{9K}+\frac{2\times\mu}{6}\left[\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right)\right]\right\}\sigma_0 \quad (8)$$

and

$$\epsilon_z=\frac{1+\mu}{3K}\sigma_0-\epsilon_x \quad (9)$$

Equations (8) and (9) are deformation patterns of boundary-nonrestrained sealing materials.

DEFORMATION PATTERN OF SEALING MATERIAL BEING BOUNDARY-RESTRAINED

In the case of viscoelastic sealing material contacting the cement mortar infill directly, when the latter breaks, the former remains bonded to it. The sealing material becomes a wide slab with one end and bottom fixed. The upper surface is free, and the other end is subjected to a uniform load $\sigma_0(t)$ as shown in Figure 7. Because the outward displacement of the slab body is constant everywhere along the z-direction, load on the sealing material can be regarded as homogeneous.

As viscoelasticity and elasticity are similar mechanically, we approach it first as an elastic body and then as a viscoelastic body. By introducing the "Principle of Correspondence of Viscosity with Elasticity"[5,6] the solution of this problem can be obtained.

Principle of Correspondence of Viscosity with Elasticity

The constitutive relation of body (H-N)-(H/N)-(H/N) is given in equations (6) and (7), while that of general linear viscoelastic body may be expressed as:

$$P(D)S_{ij}=2Q(D)e_{ij}$$

$$\Phi(D)\sigma_m=3\Psi(D)E_m$$

Here P(D), Q(D), Φ(D), and Ψ(D), represent polynomials of operator D.

The stress and strain of a viscoelastic body must satisfy, as an elastic body does, the equilibrium equation, geometric equation, and boundary conditions other than the constitutive equation, and they all relate to the time variable.

Applying Laplace transformation to the basic mechanics of a viscoelastic body, we get, under undisturbed initial conditions, the following:

$$\left.\begin{array}{l}\bar{\sigma}_{ij,j}+\bar{f}_i(\chi_k,S)=o,\\ \bar{\epsilon}_{ij}=\tfrac{1}{2}(\bar{U}_{ij}+\bar{U}_j,i),\\ P(S)\bar{S}_{ij}=2Q(S)\bar{e}_{ij},\\ \Phi(S)\bar{\sigma}_m=3\Psi(S)\bar{\epsilon}_m,\\ \bar{T}_i(\chi_j,S)=\bar{\sigma}_{ij}\eta_j\end{array}\right\} \quad (10)$$

Here tensor symbols and hereditary summation stipulation are adopted.

The subscript after the comma indicates partial derivatives with respect to the corresponding coordinate.

$\bar{S}_{ij},\bar{e}_{ij},\bar{f}_i(\chi_k,s)$ and $\bar{T}_i(\chi_k,s)$

represent respectively the Laplace transforms of deviatoric tensor of stress s_{ij}, of deviatoric tensor of strain e_{ij}, of body force $f_i(\chi_k,t)$, and of surface force $T_i(\chi_k,t)$.

If we take Q(s)/P(s)=G (shear modulus), Ψ(s)/Φ(s)=K, body force as $\bar{f}_i(x_k,s)$, surface force as $\bar{T}_i(x_k,s)$, equation (10) will correspond to the problem of an elastic body with the same geometry as the original viscoelastic body.

In case of "proportional loading" on a body:

$T_i(\chi_k,t)=T^0_i(\chi_k)f_1(t)$,
$f_i(\chi_k,t)=f_i^0(\chi_k)f_2(t)$

Applying Laplace transformation, we have:

$\bar{T}_i(\chi_k,s)=T^0_i(\chi_k)\bar{f}_1(s)$,
$\bar{f}_i(\chi_k,s)=f_i^0(\chi_k)\bar{f}_2(s)$

After transformation, the result of the space distribution of surface force and body force of the corresponding elastic body remains the same. Therefore, under the similar condition, keeping space form unchanged, the solution we get by aid of the Theory of Elasticity can be applied to the relative viscoelastic problem. By replacing only the surface force and body force by their corresponding Laplace transforms, G by Q(s)/P(s), and K by $\Psi(s)/\Phi(s)$, we get the Laplace transform which is the solution to the corresponding viscoelastic problem. By inverse transformation the solution to the viscoelastic problem itself emerges.

Deformation Pattern of an Elastic Body Being Boundary-Restrained

With reference to Figure 7, as the dimensions in y-direction (perpendicular to the paper surface) is comparatively large, we can take it as a strain problem in plane. As "a" (seam breadth) is very small, when z is unchanged, the stress component $\sigma\chi$ in any point of an elastic body may as well be taken as a linear function of x. Now assume:

$\sigma_x=(\chi+m)f(z)$.

The stress function will be:

$g(\chi,z)=(\chi+m)\iint f(z)dzdz+zf_1(x)+f_2(x)$

Putting it into double harmonic equations where the body force is constant:

$$\frac{a^4g}{at^4}+2\frac{a^4g}{ax^2az^2}+\frac{a^4g}{az^4}=0$$

we have,

$$\frac{d^4f_1(x)}{dx^4}=0$$

$$\frac{d^4f_2(x)}{dx^4}=0$$

$$\frac{d^2f(z)}{dz^2}=0.$$

Therefore,

$f(z)=Az+B$
$f_1(X)=Cx^3+Dx^2+Ex+F$
$f_2(X)=Gx^3+Hx^2+Ix+J$.

As the linear parts of $f_1(x)$, $f_2(x)$ have no effect upon the value of stress, it is negligible.

Assume z is the component of a body force in z direction, then:

$$\sigma_x=(x+m)f(Az+B) \quad (11)$$

$$\sigma z=z(6cx+2D)+6Gx+2H-Zz \quad (12)$$

$$T_{xz}=-\left(\frac{Az^2}{2}+Bz\right)+L-(3cx^2+2Dx) \quad (13)$$

Introducing the following boundary conditions: *(Figure 7)*

1° on side OM:

$z=0$

$$\int_0^a \sigma zdx=0$$

$$\int_0^a Txzdx=0$$

$$\int_0^a \sigma z\left(x-\frac{a}{2}\right)dx=0.$$

2° on side MN:

$x=a$

$$\int_0^h Txzdz=0$$

$$\int_0^h \sigma x\left(z-\frac{h}{2}\right)dz=0$$

$$\int_0^h \sigma_xdz=\sigma_0(t)h$$

$$\int_0^h Ezdz=0.$$

3° on side OP:

$x=0$

$$\int_0^h Ezdz=0.$$

By calculating, the coefficients in (11), (12), (13) are determined:

$A=0$
$G=0$
$H=0$

$$m=-\frac{(3h^2+Aa^2\mu)\sigma_0(t)+3a^2hz}{6a\mu\sigma_0(t)+3ahz}$$

$$B=\frac{a(6\mu\sigma_0(t)+3hz)}{2a^2\mu-3h^2}$$

$$C=\frac{a\mu(2\mu\sigma(t)+hz)}{h(2a^2\mu-3h^2)}$$

$$D=-\frac{[4a^2hz+6h^2\sigma_0(t)]\mu+8a^2\sigma_0(t)\mu^2+3h^2z}{2h(2a^2\mu-3h^2)}$$

$$L=-\frac{4a^3\sigma_0(t)\mu^2+2a^3hz\mu+6ah^2\sigma_0(t)\mu+3ahz^3}{2h(2a^2\mu-3h^2)}$$

Substituting them in (11), (12), and (13), we have:

$$\sigma_x=\frac{(6ahx-4a^2h)\mu_0\sigma(t)+3h^2axZ-(3h^3\sigma_0(t)+3a^2h^3z)}{\eta(2a^2\mu-3h^2)}$$

$$\sigma_z=\frac{(12ax-8a^2)\mu^2\sigma_0(t)z+(6ahzx-6a^2hz-6h^2\sigma_0(t)uz}{\eta(2a^2\mu-3h^2)}$$

$$T_{xz}=\frac{1}{2h(2a^2\mu-3h^2)}[(16a^2t-12ax^2-Aa^3)\mu^2\sigma_0(t)+(12h^2\sigma_0(t)x-12a\sigma_0(t)x^2-12ah\sigma_0(t)z-6ah^2\sigma_0(t)+8a^2hzx-6ahzx^2-2a^3hz)\mu+3ah^3z-6ah^2Zz]$$

$$\epsilon_x=\frac{1-\mu^2}{E}\left(\sigma_x-\frac{\mu}{1-\mu}\sigma_z\right)$$

$$\epsilon_z=\frac{1-\mu^2}{E}\left(\sigma_z-\frac{\mu}{1-\mu}\sigma_x\right).$$

Deformation Pattern of a Viscoelastic Body Being Boundary-Restrained

Neglecting the less significant body forces z, we get the strain pattern of an elastic body:

$$\epsilon_x=6(ax-4a^2)\frac{(1-\mu^2)\mu\sigma_0(t)}{E(2a^2\mu-3h^2)}-\frac{(3h^2)(1-\mu^2)}{(2a^2\mu-3h^2)E}\sigma_0(t) \quad (14)$$
$$+\frac{(8a^2z-12axz)}{\eta}\frac{(1+\mu)\mu^3}{E(2a^2\mu-3h^2}\sigma_0(t)+6hz\frac{(1+\mu)\mu^2}{E(2a^2\mu-3h^2)}\sigma_0(t).$$

$$\epsilon_z=\frac{(12axz-8a^2z}{h}\cdot\frac{(1-\mu^2)\mu^2\sigma_0(t)}{E(2a^2\mu-3h^2)}-6hz\cdot\frac{(1-\mu^2)\mu\sigma_0(t)}{E(2a^2\mu-3h^2)}$$
$$-(6ax-4a^2)\frac{(1+\mu)\mu^2\sigma_0(t)}{E(2a^2\mu-3h^2)}-3h^2\frac{(1+\mu)\mu}{E(2a^2\mu-3h^2)}\sigma_0(t). \quad (15)$$

By Theory of Elasticity, we know:

$$E=\frac{9KG}{3K+G},\quad \mu=\frac{3K-2G}{6K+2G}$$

Provided $p=6a^2-18h^2$, $q=4a^2+bh^2$, we easily get the following:

$$\frac{1-\mu^2}{E(2a^2\mu-3h^2)}=\left(\frac{3K}{2G}+2\right)\times\frac{1}{PK-qG}$$

$$\frac{(1-\mu^2)\mu}{E(2a^2\mu-3h^2)}=\left(\frac{3K}{4G}-\frac{9K}{4}\cdot\frac{1}{3K+G}\right)\times\frac{1}{pK-qG}$$

$$\frac{(1-\mu^2)\mu^2}{E(2a^2\mu-3h^2)}=\left[\frac{9K^2}{8G}-\frac{B7K}{8}\cdot\frac{1}{3K+G}+\frac{135K^2}{8}\cdot\frac{1}{(3K+G)^2}+2\right]\times\frac{1}{pK-qG}$$

$$\frac{(1+\mu)\mu}{E(2a^2\mu-3h^2)}=\left(\frac{3K}{2G}-1\right)\times\frac{1}{pK-qG}$$

$$\frac{(1+\mu)\mu^2}{E(2a^2\mu-3h^2)}=\left[\frac{3K}{4G}-\frac{27K}{4(3K+G)}+1\right]\times\frac{1}{pK-qG}$$

$$\frac{(1+\mu)\mu^3}{E(2a^2\mu-3h^2)}=\left[\frac{3K}{8G}+\frac{9K}{3K+G}-\frac{2A3K^2}{8(3K+G)^2}\right]\times\frac{1}{pK-qG}.$$

As $\frac{G}{3K}\ll 1$, $\frac{8G}{pK}\ll 1$,

develop $\frac{1}{3K+G}$, $\frac{1}{pK-8G}$ into power series.

Replacing (14) and (15) by the foregoing expressions, we have:

$$E_x=\frac{\sigma_0(t)}{pK}\left[\frac{B_1}{G}+B_2G+B_3\right] \quad (16)$$

$$E_z=\frac{\sigma_0(t)}{pK}\left[\frac{C_1}{G}+C_2G+C_3\right] \quad (17)$$

where

$$B_1=(60x-4a^2)\times\frac{3K}{4}-\frac{9h^2K}{2}+\frac{3K(8a^2z-12axz)}{8h}+\frac{9hKz}{2}$$

$$B_2=(6ax-4a^2)\left(\frac{1}{AK}-\frac{3q}{AK}\right)-\frac{6h^2q}{pK}+\frac{8a^2z-12axz}{h}\times\left(\frac{43q}{BpK}+\frac{5}{AK}\right)+\frac{3}{AK}-\frac{5q}{ApK}$$

$$B_3=(6ax-4a^2)\left(\frac{3q}{Ap}-\frac{3}{A}\right)-\frac{9h^2q}{2p}+\frac{1}{h}\times(8a^2z-12axz)\times\left(\frac{43}{8}+\frac{23q}{4p}\right)+3hz\left(\frac{3q}{2p}-\frac{5}{2}\right)$$

$$C_1=\frac{3K}{2h}\times(3axz-2a^2z)-\frac{9Khz}{2}+\frac{3K(2a^2-3ax)}{2}-\frac{9h^2K}{2}$$

$$C_2=\frac{1}{h}\times(2a^2z-3axz)\left(\frac{1}{6K}+\frac{8}{6pK^2}+\frac{31q}{2pK}\right)+3hz\left(\frac{3q}{2K}-\frac{1}{2K}\right)+(2a^2-3ax)\left(\frac{3}{2K}-\frac{5q}{2pK}\right)+\frac{3h^2q}{pK}$$

$$C_3=\frac{1}{h}\times(3axz-2a^2z)\left(1+\frac{3q}{2p}+\frac{q}{2pK}\right)-3hz\left(\frac{3q}{2p}-\frac{3}{2}\right)+(2a^2-3ax)\left(\frac{3q}{2p}-\frac{5}{2}\right)+3h^2\left(1-\frac{3q}{2p}\right).$$

In accordance with the Correspondence Principle of Viscosity with Elasticity, it is obvious that the external load $\sigma_0(t)$ is referred to "proportional loading."

Substitute G in (16), (17) with:

$$G=\frac{1}{\frac{1}{E_1}+\frac{1}{\eta_1 s}+\frac{1}{E_2+\eta_2 s}+\frac{1}{E_3+\eta_3 s}}$$

$$\left(\text{viz. } G=\frac{B_3s^3+B_2s^2+B_1s}{\alpha_3s^3+\alpha_2s^2+\alpha_1s+\alpha_0}\right)$$

$$\frac{1}{G}=\frac{1}{E_1}+\frac{1}{\eta_1 s}+\frac{1}{E_2+\eta_2 s}+\frac{1}{E_3+\eta_3 s}$$

Replace $\sigma_0(t)$ with σ_0/s. We get the Laplace transform of viscoelasticity written in fractions with separated terms:

$$\overline{E}_x=\frac{\sigma_0}{pK}\left\{B_1\left[\frac{1}{E_1s}+\frac{1}{\eta_1s^2}+\frac{1}{s(E_2+\eta_2s)}+\frac{1}{s(E_3+\eta_3s)}\right]+B_2\left[\frac{s}{(s-A_1)(s-A_2)}+\frac{1}{(s-A_3)(s-A_4)}\right]+\frac{B_3}{s}\right\}$$

$$\overline{E}_z=\frac{\sigma_0}{pK}\left\{C_1\left[\frac{1}{E_1s}+\frac{1}{\eta_1s^2}+\frac{1}{s(E_2+\eta_2s)}+\frac{1}{s(E_3+\eta_3s)}\right]+C_2\left[\frac{s}{(s-A_1)(s-A_2)}+\frac{1}{(s-A_3)(s-A_4)}\right]+\frac{C_3}{s}\right\}.$$

Applying inverse Laplace transform [7], we get the real solutions to the original viscoelasticity,

$$E_x=\frac{\sigma_0}{pK}\left\{B_1\left[\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right)\right]+B_2\left[\frac{1}{A_1-A_2}(A_1e^{A_1t}-A_2e^{A_2t})+\frac{1}{A_1-A_2}(e^{A_1t}-e^{A_2t})\right]+B_3\right\}$$

$$E_z=\frac{\sigma_0}{pK}\left\{C_1\left[\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right)\right]+C_2\left[\frac{1}{A_1-A_2}(A_1e^{A_1t}-A_2e^{A_2t})+\frac{1}{A_1-A_2}(e^{A_1t}-e^{A_2t})\right]+C_3\right\}$$

where,

$$A_1=\frac{1}{2\eta_1\eta_2}\left[-(\eta_1E_1+\eta_1E_2+\eta_2E_1)+\sqrt{(\eta_1E_1+\eta_2E_2+E_1\eta_2)^2-A\eta_1\eta_2E_1E_2}\right]$$

$$A_2=\frac{1}{2\eta_1\eta_2}\left[-(\eta_1E_1+\eta_2E_1+\eta_1E_2)+\sqrt{(\eta_1E_1+\eta_1E_2+\eta_2E_1)^2-A\eta_1\eta_2E_1E_2}\right]$$

$A_3=A_1$, $A_A=A_2$

(Note: According to the data from actual measurement we know the influence of the second Kelvin element of the rheological model in Figure 5 is slight, and G, too, has little influence on the result. Therefore, for simplicity, the second Kelvin element is omitted in the G expression, but when calculating 1/G, the second Kelvin element is not overlooked.)

CONCLUSION

The analysis provides the deformation pattern of sealing material under two boundary conditions, restrained and non-restrained. Of special interest is the deformation pattern in x-direction, perpendicular to the joint direction.

1. Boundary non-restrained:

$$Ex_1=\sigma_0\left\{\frac{2-\mu}{6}\left[\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right)\right]+\frac{1+\mu}{9K}\right\} \quad (18)$$

2. Boundary restrained:

$$Ex_2=\frac{\sigma_0}{pK}\left\{B_1\left[\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}t}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right)\right]+B^2\left[\frac{1}{A_1-A_2}(A_1e^{A_1t}-A_2e^{A_2t})+\frac{1}{A_1-A_2}(e^{A_1t}-e^{A_2t})\right]+B_3\right\} \quad (19)$$

Simplifying equations (18) and (19) we get

$$Ex_1=\sigma_0\left[\propto\left(\frac{2-\mu}{6}\right)+\frac{1+\mu}{9K}\right]$$

$$Ex_2=\frac{\sigma_0}{pK}(\propto B_1+r).$$

where,

$$\propto=\frac{1}{E_1}+\frac{t}{\eta_1}+\frac{1}{E_2}\left(1-e^{-\frac{E_2}{\eta_2}}\right)+\frac{1}{E_3}\left(1-e^{-\frac{E_3}{\eta_3}t}\right).$$

3. Comparing results from (1) and (2) using actually measured data in the relative expressions, we get:

$A_1=-0.033913$,

$A_2=-6.200341$,

$e^{A_1t}=(0.966556)^t$,

$e^{A_2t}=(0.002029)^t$.

With increased time, we know $r<<\alpha B_1$. On the other hand, $(1+\mu)/9K$ is much smaller than $\propto\left(\frac{2-\mu}{6}\right)$. Therefore:

$$Ex_1\approx\propto\sigma_0\left(\frac{2-\mu}{6}\right)$$

$$Ex_2\approx\frac{\propto B_1\sigma_0}{pK}$$

$$\frac{Ex_1}{Ex_2}\approx\frac{2-\mu}{6}\Big/\frac{B_1}{pK}.$$

Where:

$$B=\frac{3K}{4}(6ax-4a^2)-\frac{9h^2K}{2}+\frac{3K(8a^2z-12axz)}{8h}+\frac{9hKz}{2}$$

$p=6a^2-18h^2$.

In China, we take $a=h$, $\mu=0.3$. On the cross-section where $x=a$,

1) If $z=0$, (that is, on the upper surface of the sealing material), we have:

$\frac{B_1}{pK}=\frac{1}{4}$, then, $\frac{Ex_1}{Ex_2}\approx1.13$

2) If $z=h/2$, we have:

$\frac{B_1}{pK}=\frac{1}{8}$, therefore, $\frac{Ex_1}{Ex_2}\approx2.26$

3) If $z=\frac{3h}{A}$, we get:

$\frac{B_1}{pK}=\frac{1}{16}$. Therefore $\frac{Ex_1}{Ex_2}\approx4.52$

4) If $z=h$ on the bottom of the sealing material, we have:

$\frac{B_1}{pK}=0$.

The ratio Ex_1/Ex_2 becomes a very large value.

$\left(\text{In this case, we consider } \frac{Ex_1}{Ex_2}\approx\frac{\propto(2-\mu)}{6}\Big/\frac{\tau}{pK}\right).$

These results show that on the surface of the sealing material along the co-axial direction, deformation, whether restrained or non-restrained, is almost equal. On the bottom of the sealing material, when it is restrained, almost no deformation occurs, which agrees with common sense.

Common sense tells us that the deformation ability of a sealing material depends largely on the deformation ability of the bottom of a seal to deform. The bottom breaks first, then the whole body breaks.

Let's consider the average ratio of strain within the interval of z=h/2~h. We have:

$$\frac{\frac{1}{h/2}\int_{h/R}^{h} Ex_1 dz}{\frac{1}{h/2}\int_{h/2}^{h} Ex_2 dz} = \frac{\frac{2-\mu}{6}\times\frac{1}{2}}{\frac{3}{8\times 12}} \approx 4.5$$

On examining the ratio of average strain where z=3h/4~h, we have,

$$\frac{\frac{1}{h/4}\int_{3h/4}^{h} Ex_1 dz}{\frac{1}{h/4}\int_{3h/4}^{h} Ex_2 dz} = \frac{\frac{2-\mu}{6}\times\frac{1}{4}}{\frac{3}{32\times 12}} \approx 9.0$$

It is reasonable to take the average ratio of strain. From the above data, when a disbonding layer is put between the sealing material and cement mortar infill, the deformation ability of the sealing material will be increased greatly, up to five or ten times or more.

At certain places in China where these ways of jointing were put into practice, measurements of the two elongation rates were taken on the spot. The results are shown in Table 4.

Kind of Sample	Sample No.	Extension Without Disbonding Layer		Extension With Disbonding Layer	
		(mm)	(%)	(mm)	(%)
I	H2−1	26	173	47	313
	H2−2	30	200	52	346
	H2−3	24	160	41	270
II	S1−1	21	140	41	270
	S1−2	18	120	36	240
	S1−3	17	113	31	206
III	04−1	23	153	433	286
	04−2	29	193	51	340
	04−3	21	140	38	253

Table 4 Extension of sealing material

The table shows that with a disbonding layer the rate of elongation is about doubled. This corresponds to $\epsilon x_1/\epsilon x_2 \approx 2$, which probably is due to the poor friction of the disbonding layer, a problem of construction technology. However, for us to improve such kind of construction technology is necessary.

SUMMARY

A waterproof sealing material, whose ability to accommodate joint movement depends upon its viscoelastic properties, is characterized by its elongation potential.

The elongation potential is greatly dependent upon the degree of restraint to deformation within the joint.

Based on experimental data, this paper derives a mathematical model for a viscoelastic sealing strip with due allowance for the degree of restraint.

It is demonstrated that the use of a disbonding layer between the sealing strip and a cement mortar joint infill will increase the movement capability of the seal more than 4 to 5 times.

CONTENTS

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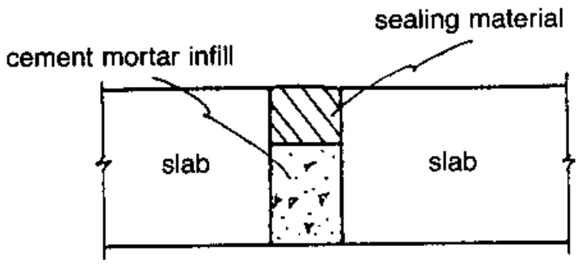

Figure 1 *Joint before deformation*

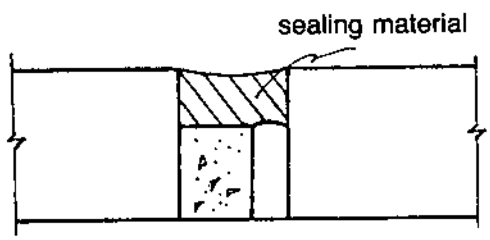

Figure 2 *Joint after deformation*

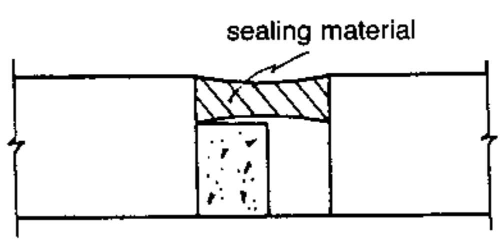

Figure 3

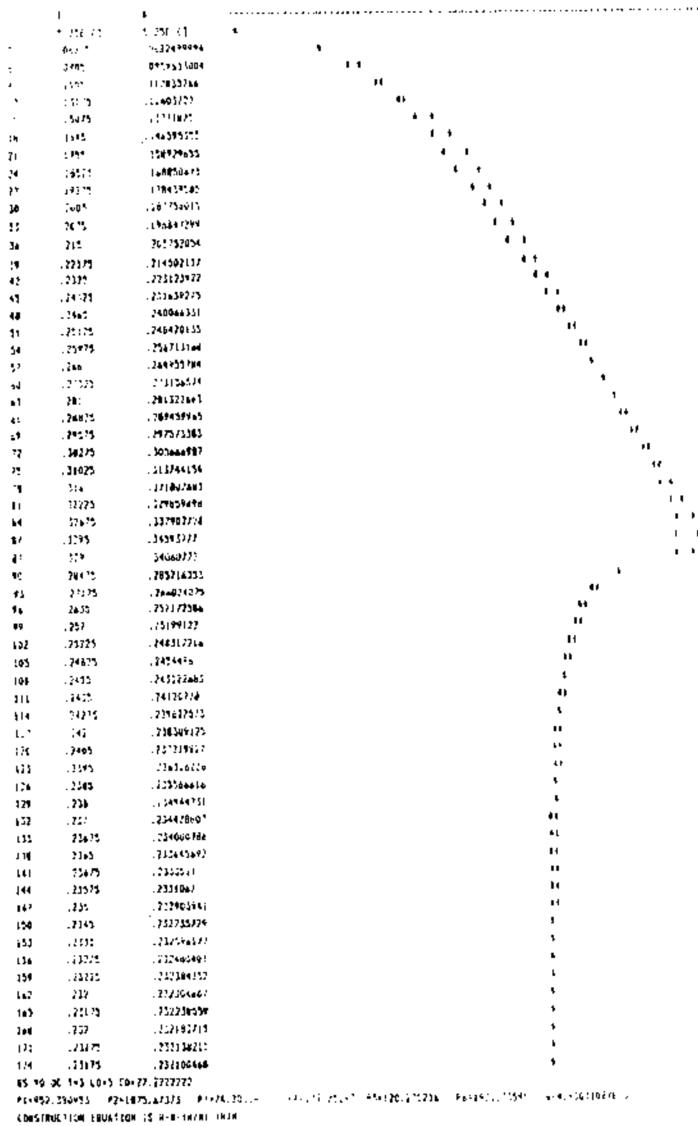

Figure 4

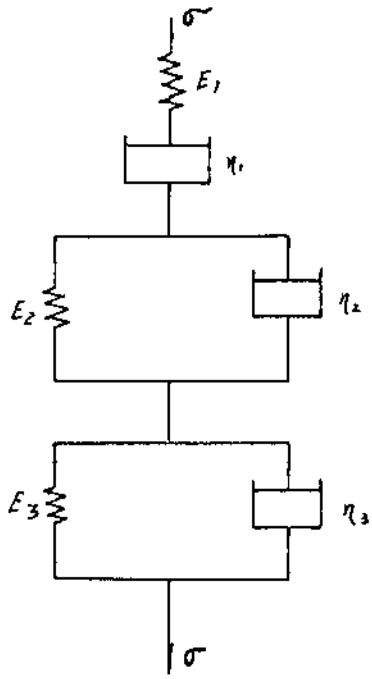

Figure 5

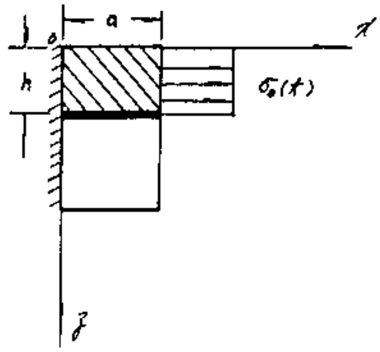

Figure 6

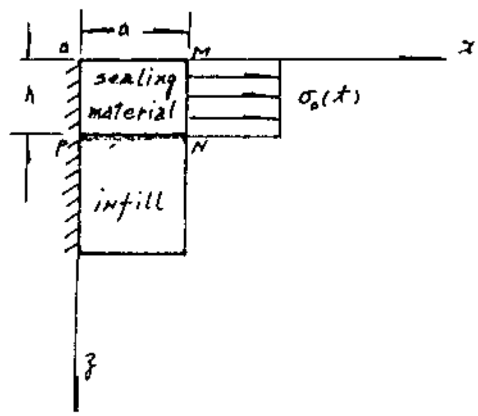

Figure 7