FINITE ELEMENT MODELLING OF REINFORCED BITUMEN MEMBRANES

G. WAYNE BRODLAND, MARCUS J. DELL and ERIC F.P. BURNETT
University of Waterloo
Waterloo, Canada

CHRIS GAILEY
Crucial Technologies, Inc.
Toronto, Canada

This paper outlines a finite element software package that has been developed for the analysis of the mechanical performance of reinforced bitumen membranes. The software permits the incorporation of both highly nonlinear and visco-elastic material properties. It also allows modelling of the sequence of failure and identifies the factors (e.g., debonding) that lead to total membrane system failure. Non-adhered, single-ply as well as fully-adhered, multi-ply membrane systems can be analyzed. The finite element results compare well with laboratory experiments. Such analyses provide considerable insight into the mechanics of membrane response and failure. The software is also a valuable development tool.

KEYWORDS

Finite element analysis, in-plane response, modelling, multiply membranes, nonlinearity, polymer modified bitumen, reinforced roofing membranes, visco-elasticity.

INTRODUCTION

Analytical procedures, such as those of Marijs and Bonafont,[1] for reinforced membranes have provided a useful complement to laboratory tests of roofing membranes and other membrane systems. In particular, they enable the stresses and strains produced in the membrane system under various loading conditions to be estimated. This, in turn, provides insight into the response of the membrane and may prompt design improvements. In addition, such analytical tools provide a method for assessing the in-plane behavioral effects of possible design alterations without the high cost of prototype fabrication and laboratory testing.

The use of polymer-modified bitumens and of reinforcements, both of which can have complex behavioral characteristics and can withstand large strains, has considerably complicated the analysis. In particular, these materials can exhibit highly nonlinear, and sometimes visco-elastic, material properties, making accurate analyses using traditional mathematical methods virtually impossible. Furthermore, it is becoming increasingly clear that nonlinear and viscous mechanical properties must be included if the analysis is to be used as either a developmental tool or a performance predictor.

The purpose of this paper is to discuss a nonlinear finite element software package, REMA (Reinforced Membrane Analysis), that the authors have developed to model nonlinear, visco-elastic, reinforced, membrane systems. The sequence of membrane component failures and debondings which lead to total membrane system failure can also be modelled. This paper briefly outlines the physical and theoretical background of the software. An example shows the utility of the program. Possible applications of the program are also discussed.

The software has been specifically written for use on a microcomputer to allow for low cost, easily accessible operation. A computer with a 80386 processor (16 Mhz or better) and a math co-processor is, however, recommended.

PHYSICAL BASIS OF FORMULATION

In most reinforced bituminous membranes under in-plane loading, the reinforcements carry tensile loads while the bitumen layers carry tension and shear loads. Since membranes are thin, through-thickness stresses are negligible. Also, since such membranes are usually wide, the in-plane strains normal to the direction of loading are negligible, only displacements in the direction of loading need be considered in the analysis. For a membrane system loaded in the x-direction, (see Figure 1) this implies two conditions, $\sigma_y = 0$ and $\epsilon_z = 0$, which are important in simplifying the stress-strain relationships. These analytical simplifications considerably reduce the complexity of the resulting numerical formulation.

FINITE ELEMENT FORMULATION

Three categories of information are required before the software package can be run; namely, material properties, the geometric configuration of the membrane, and the loading to which the membrane will be subjected. Since typical roofing membrane elastomers and reinforcements can have highly nonlinear, visco-elastic properties, the material properties are an important aspect of the analysis.

It has been found useful to model the visco-elastic behavior of modified bitumens using a Kelvin-Voigt model[3] (see Figure 2). Thus, the total local shear stress, τ, in a modified bitumen is the sum of its elastic, τ_e, and viscous, τ_v, components given in terms of its current local shear strain, γ, and time rate of shear, $\dot{\gamma}$, by:

$$\tau_e = A\gamma + B\gamma|\gamma| + C\gamma^3 \tag{1}$$

and

$$\tau_v = D\dot{\gamma} + E\dot{\gamma}|\dot{\gamma}| + F\dot{\gamma}^3. \tag{2}$$

The empirical constants A through F are selected to best represent the actual material behavior, as determined, for example, by laboratory tests. The approximate mechanical behavior of an SBS polymer-modified bitumen tested in pure shear at −18°C bitumen is shown in Figure 3. At this temperature, there is no discernable viscous deformation during the time scale of interest.

Reinforcements typically exhibit behavior which is even more nonlinear, and are therefore modelled as being piecewise quadratic, i.e.,

$$\sigma_i = P_i + Q_i\epsilon + R_i\epsilon^2, \ \epsilon_{i-1} \leq \epsilon < \epsilon_i. \tag{3}$$

This empirical expression permits an accurate representation of a wide range of highly nonlinear reinforcement behaviors. For example, the approximate mechanical behavior of a bitumen impregnated polyester mat, tested at −18°C, is shown in Figure 4.

The next category of information required to run the software is the geometry of the membrane; its materials, their thicknesses and their ordering.

The third category of required information is the loading to which the membrane will be subjected. It is useful to describe the external loads or displacements as linearly varying quantities over each of the four membrane faces, with provision for initial debonding along part of each face.

During input of the load characteristics, the horizontal spacing of the nodes must be defined. It is useful to do this using a fourth-order polynomial. The software uses the polynomial in conjunction with the thickness of the material layers to construct the mesh which divides the membrane. Discretization, as the name implies, is the process whereby the membrane is divided into discrete 'elements.' Rectangular elements, as shown in Figure 5, with linear shape functions, are used. Using this discretization, the program automatically assigns elements and node numbers and determines element connectivity. It is important to select an equation that places small elements in regions where stress gradients are high, such as at discontinuities and jaw edges.

New nodes are required whenever separation or debonding occurs between adjacent elements. In order to automatically include such new nodes without unduly increasing the bandwidth of the equations that must be solved, an automatic mesh generator was written and incorporated into the analysis program.

The actual finite element procedure is shown schematically in Figure 6. After entering of the material, geometry and load information described earlier, the user initiates this portion of the program.

After reading the required information, developing the element mesh and assigning nodes, the program sets up the stiffness matrix, **K**. This matrix describes how all the nodal displacements, **u**, are related to the applied nodal forces, **f**, i.e.,

$$[\mathbf{K}]\,\{\mathbf{u}\} = \{\mathbf{f}\}. \tag{4}$$

The terms within this matrix depend on the shape and size of the individual elements, and on the stiffness of their materials.[2] Strain and strain rate dependent material properties are calculated for each element individually, based on the strain state of the centroid of the element.

Since the matrix, **K**, is large and contains many zeros, it is important to store it in a compact form. A highlight form was used in which only one-half of the symmetrical matrix is calculated and stored. This storage economy was crucial to making the program microcomputer-compatible.

Equation 4 was partitioned according to the degrees of freedom for which displacements were known, and a half-band Gauss solver was used to solve the resulting reduced matrix equation.

Since nonlinear material properties are used and since debonding may even change the geometry of the membrane, it is necessary to check the validity of the resulting calculations. This is done by calculating new nodal forces based on the new nodal positions and new elemental material properties. These nodal forces are included in the load vector, **f**, for addition to the incremental external loads associated with the next iteration.

After calculating the stresses in each element, the program determines whether any of the calculated stresses exceed its respective debonding values. If more than one element has a stress that exceeds its allowable limit, the program determines which element exceeds its limit by the greatest percentage. This element is then debonded, either in shear or in tension and new nodes are added as required. Based on the new nodal configuration, the element stresses are recalculated to allow for stress redistribution. The stresses are then compared again to the allowable maximums to determine whether further debonding is required. If so, additional elements are debonded using this same procedure.

If additional incremental load steps remain to be applied, they are read and applied to the deformed membrane.

SAMPLE ANALYSIS OF A MULTI-PLY MEMBRANE

The following analysis of a reinforced, polymer-modified bitumen, fully-adhered, roof membrane illustrates some of the features discussed previously. The composition is like that of a cap sheet and base sheet bonded together. The system being modelled consists of a glass mat-reinforced base sheet bonded to a rigid substrate and a cap sheet with polyester mat-reinforcement bonded to the top of the base sheet (see Figure 7). Under the center of the sample there is a joint in the substrate. Since the geometry and loading are symmetrical, only one half needs to be modelled. The two sections of substrate are separated using displacement control. The sample analysis is for a temperature of −18°C.

The material properties for the bitumen and the polyester are those shown in Figures 3 and 4, respectively. The glass mat was modelled as having a linear response with a slope of 900 N/mm up to a tensile failure strength of 30 N/mm.

Figure 5 illustrates the membrane prior to load application. For clarity, the vertical axis is enlarged by a factor of 20, and the displacements in the horizontal direction are magnified by a factor of 10. Note that there is a high concentration of elements near the center line (i.e., directly above the substrate joint). Figures 8 and 9 illustrate, respectively, the element configuration just prior to and directly after rupture of the glass mat. Note that when the glass mat fails, the bitumens directly above it also fails. When the stresses in the polyester mat become excessive and exceed the allowable stresses, it also fails, as does the bitumen that overlies it.

Figure 10 illustrates the highly nonlinear load-deflection curve generated by the finite element model. Examination of this numerical data shows that key features of the load-deflection curve are caused by rupture of the reinforcements at different locations. For comparative purposes, a typical load-deflection curve determined by laboratory testing for a similar membrane system is included in Figure 10.

It is clear that not only can the finite element analysis predict the complex qualitative features of the multi-ply membrane system, but it can also accurately predict key quantitative values.

The software package was written in Microsoft QuickBASIC. The run time for this example was approximately 15 minutes, including printing of the deformed meshes and the load-deflection plots, when run on a 16 Mhz, 80386-based personal computer with a math co-processor.

COMMENTARY AND CONCLUSIONS

To date, REMA has been used largely as an analytical tool to model in-plane mechanical testing, specifically tensile and joint-bridge testing of modified bitumen membranes. In fact, it is only by interactive comparison of analytical and experimental results that the actual properties of reinforcement, coated and constrained in bitumen, can be derived. The mechanical behavior of a membrane ply cannot be established by merely adding the properties of the individual materials. This is because the tensile response of bitumen-coated reinforcement is very different from the tensile response of the reinforcement alone. By matching the experimental and finite element results, it is possible to determine a force-displacement relationship for bitumen-coated reinforcement.

Both of the commercial partners involved in the development of REMA are using it for development and marketing purposes. Its use for product development is particularly important because it enables a proposed new design to be evaluated without the time and cost of a trial production run and subsequent laboratory testing. Although in some instances one may have reservations about the absolute validity or accuracy of an answer, the relative difference between two similar designs is usually an accurate means of tracking the influence of a change to a single variable. It follows that any form of parametric analysis can readily be done using REMA. It is much less expensive and quicker to conduct "experiments" on REMA than in the laboratory or in the plant.

The use of REMA has had some unexpected spin-offs. Any membrane can be modelled, whether a roofing membrane, an air barrier, or a parking garage floor. Not only can REMA be used to model individual plies, but multi-ply systems can also be modelled. REMA enables the membrane designer to answer 'what if' questions and, for this reason alone, the time and effort spent on developing the microcomputer-based tool has been worthwhile.

ACKNOWLEDGMENTS

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[2] Logan, D.L., A First Course in the Finite Element Method, PWS Engineering, Boston, Mass., 1986.

[3] Rudin, A., The Elements of Polymer Science and Engineering, Academic Press, Orlando, Fla., 1982.

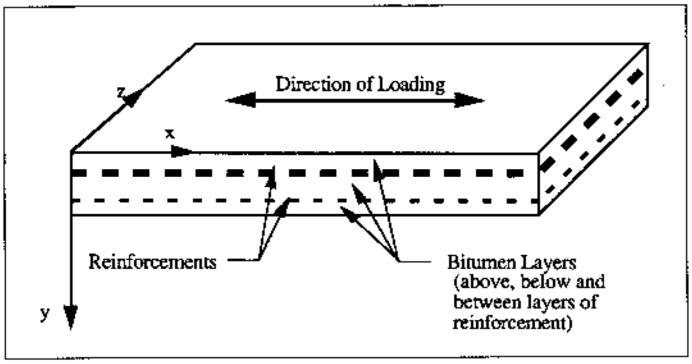

Figure 1 *A typical reinforced membrane system.*

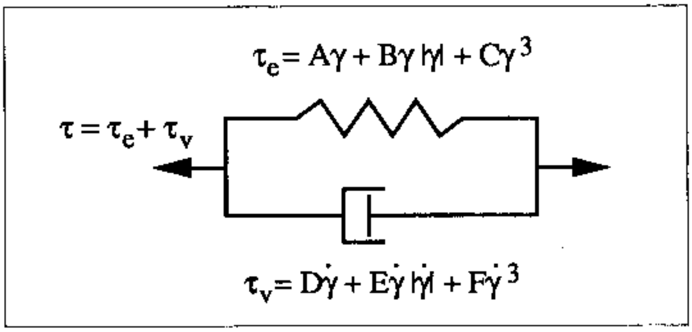

Figure 2 *The Kelvin-Voigt Model for visco-elastic behavior.*

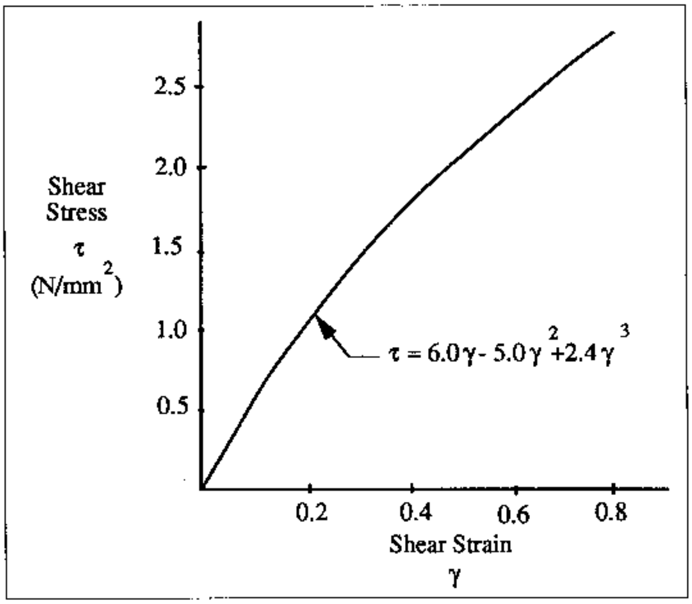

Figure 3 *Shear response of an SBS polymer-modified bitumen.*

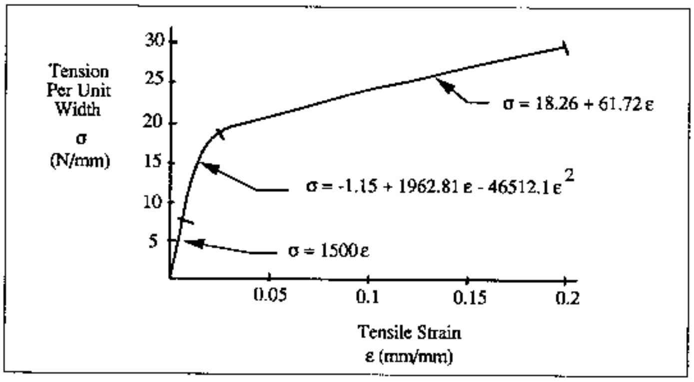

Figure 4 *Tensile response of a polyester mat reinforcement.*

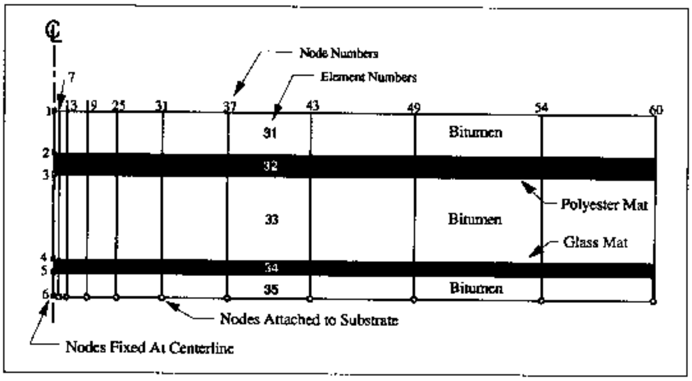

Figure 5 *Typical element layout.*

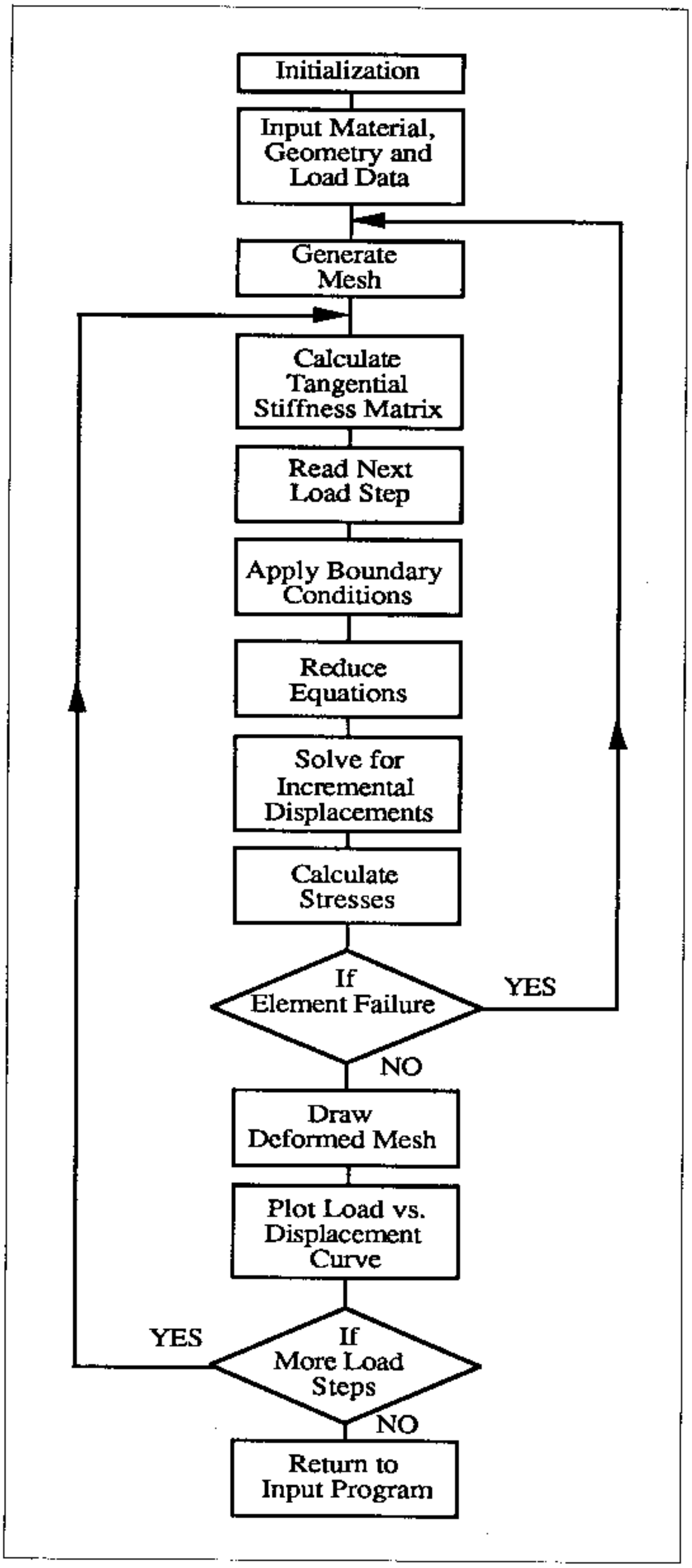

Figure 6 *General flowchart for finite element analysis.*

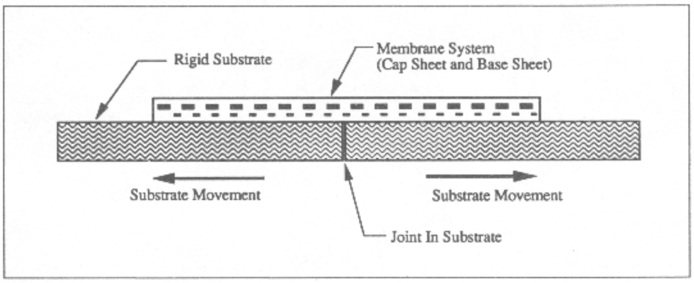

Figure 7 *Test being modelled.*

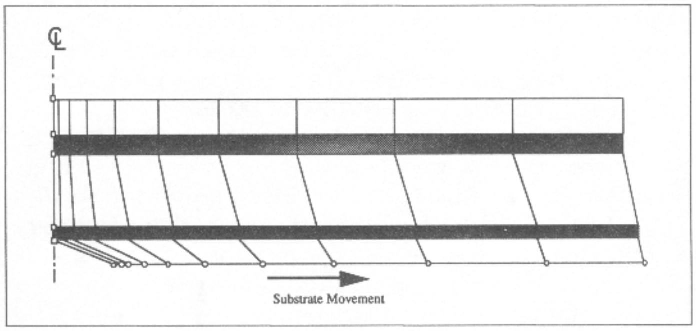

Figure 8 *Deformed mesh just prior to failure of glass mat.*

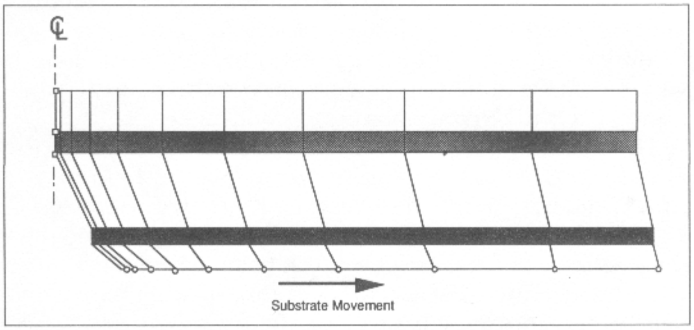

Figure 9 *Deformed mesh after failure of glass mat.*

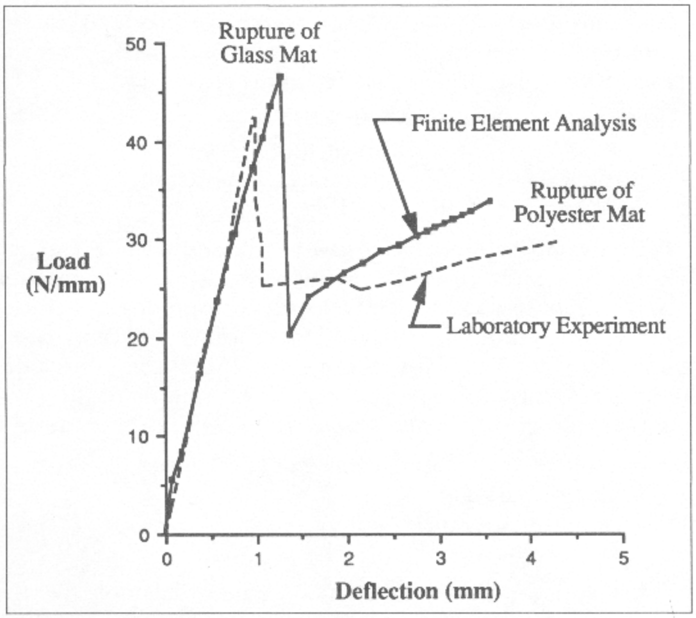

Figure 10 *Comparison of load deflection curves.*