A NEW MODEL TO DESIGN ROOFS BY MINI- AND MICRO-COMPUTERS

R. HUSSEIN

New Jersey Institute of Technology
Newark, N.J.

The roof designer of the past had to rely on experience or on experiment. When the reasons for the success of the past designs were not known, it was not possible to identify the significance of departures from them, and thus the new design inevitably became an experiment. It has been necessary, therefore, in establishing a sound basis for rational design, to seek an understanding of why various arrangements did or did not perform.

On the other hand, in the past few years, computer-aided design and drafting (CADD) had a very big impact in all of the engineering areas. Based on the widespread use of mini- and micro-computers at the present time, it is predicted that within the next decade, CADD will have an impact on design and construction similar to the impact of the automobile on transportation.

This paper presents a new model to simulate the actual behavior of any roof for the purpose of using any of the available code on mini- or micro-computers. The new model is obtained such that an actual roof system and its equivalent one behave in the structural manner. The actual value of axial, flexural and torsional rigidities are considered in developing the present model. No restrictions are imposed on boundary conditions, types of analysis and type of materials.

PROBLEM DESCRIPTION

According to the known methods for roof designs, the applied loads are resisted, in general, by: roofing, purlins and main supports such as trusses. Each of these subsystems is being designed independently from the others. Therefore, loads are transferred between the subsystems with the assumption that no integration in the roof is considered.

Considering a roof as a composite of discrete subsystems underestimates the actual stiffnesses of the whole system. This does not result in either efficient or economic structural designs. As a consequence, most of the elements, such as purlins, are often oversized.

This discretization approach usually is accompanied by intuitive decisions due to the lack of understanding of how roof systems behave. Bending of unsymmetrical sections of purlins in two planes; torsion of the purlins due to eccentric loads; considering the actual planes of bending of purlins; vibration due to a high gusty wind; considering more than one combination of loads to design all the elements, are a few among other aspects being treated approximately in the design of roof systems.

In the present work, all the subsystems of a roof are integrated into a composite to which each constitutent not only contributes its share, but whose combined action transcends the sum of the individual properties, and provides new performance unattainable by the constituents acting alone. Such approach reflects the actual behavior of a roof and leads to more efficient and economic structural designs.

A roof system is considered, in this paper, as a structural orthotropic system. This structural orthotropy is introduced by means of purlins, sheathing or corrugations. Using natural anistropic materials, such as fiber-reinforced plastics, enhances the applications of the present approach. Analytic solutions, however, for orthotropic structures are difficult and solutions for very particular cases are available.[1, 2, 3] This drawback in the available theories provides a constraint on their use in practice. At the same time, designers with limited background in the finite element method may find it of no value. Also, with the widespread use of micro-computers in solving engineering problems, new methods of solving such structural systems are needed.

THE EQUIVALENT ELEMENT FOR ROOF SYSTEMS

The actual roof element shown in Figure 1(a) is represented by four grid members connected rigidly at the corners Figure 1(b). Along the diagonals, there are two flexural beams. The flexural and torsional rigidities of the actual roof element are: D_x, D_y and D_{xy}, respectively.

When the actual roof element is subjected in turn to uniform moments M_y, M_x, and H, the rotations resulted are obtained 2 as Figures 1(c), 1(e) and 1(g):

$$\theta_1 = \frac{k\lambda M_y}{D_x} \quad (1)$$

$$\theta_2 = \frac{\nu_{yx}\lambda M_y}{D_x} \quad (2)$$

$$\theta_3 = \frac{\lambda M_x}{D_y} \quad (3)$$

$$\theta_4 = \frac{\nu_{xy}k\lambda M_x}{D_y} \quad (4)$$

$$\theta_5 = \frac{k\lambda H}{2D_{xy}} \quad (5)$$

in which

λ, $k\lambda$ = dimensions of the element;

ν_{xy}, ν_{yx} = Poisson's ratios.

On the other hand, when the equivalent model is under the resultant nodal moments as shown in Figures 1(d), 1(f) and 1(h), and by applying the known direct stiffness method of analysis, the nodal rotations are found as:

$$\theta_6 = F1/F2 \quad (6)$$

$$\theta_7 = F3/F2 \quad (7)$$

$$\theta_8 = F4/F5 \quad (8)$$

$$\theta_9 = F6/F5 \quad (9)$$

$$\theta_{10} = F7/F8 \quad (10)$$

in which

$F1 = M_y\lambda(S_1 + S_2 - S_3 - s_4)$

$F2 = (S_5 - S_6 + S_7 - S_8)(S_1 + S_2 - S_3 - S_4) - (S_9 - S_{10})(S_9 - S_{11})$

$F3 = -M_y\lambda(S_9 - S_{10})$

$F4 = k\lambda M(S_5 - S_6 + S_7 - S_8)$

$F5 = (S_1 + S_2 - S_3 - S_4)(S_5 - S_6 + S_7 - S_8) - (S_9 - S_{10})(S_{11} - S_{12})$

$F6 = -k\lambda M_x(s_{11} - S_{12})$

$F7 = 0.5\lambda H$

$F8 = -S_2 - S_4 + (S_{10}/k) + (S_{12}/S_{13})(S_{14} - (S_{15}/k))$

S_{ij} = the stiffness coefficients given in Appendix I.

PROPERTIES OF THE EQUIVALENT SYSTEM

The structural behavior of the actual roof element and its equivalent model will be the same if the following conditions are fulfilled:

$$\theta_1 = \theta_6 \quad (11)$$

$$\theta_2 = \theta_7 \quad (12)$$

$$\theta_3 = \theta_8 \quad (13)$$

$$\theta_4 = \theta_9 \quad (14)$$

$$\theta_5 = \theta_{10} \quad (15)$$

By substituting Equivalents one to 10, in Equivalents 11 to 15 and solving for the unknowns' flexural and torsional rigidities, one could obtain:

$$E_dI_d = \frac{D_xD_y\nu_{yx}\lambda r^3}{2k(D_x - \nu^2_{yx}D_y)} \quad (16)$$

$$E_cI_c = \frac{D_x\lambda(D_x - D_yk^2\nu^2_{yx})}{2(D_x - \nu^2_{yx}D_y)} \quad (17)$$

$$E_sI_s = \frac{D_xD_y(k^2 - \nu_{yx})}{2k(D_x - \nu^2_{yx}D_y)} \quad (18)$$

$$G_cJ_c = D_{xy}\lambda - \frac{D_xD_y\nu_{yx}\lambda}{D_x - \nu^2_{yx}D_y} \quad (19)$$

$$G_sJ_s = G_c J_c k \quad (20)$$

in which

E = Modulus of elasticity;

I = Moment of inertia

s, c, d = Subscripts denoting side, chord diagonal members, respectively.

NUMERICAL RESULTS

Consider the roof shown in Figure 2(a). The roof covering is ¼-inch corrugated sheets spanning 4 feet between purlins. The span of a typical truss is 37 feet whereas, the spacing is 16 feet as shown in Figure 2(b). The roof is 1/4 pitch or 9 feet high.

The loads considered acting on the roof are:

wind pressure = 20 psf on a vertical surface;

snow load = 30 psf on a horizontal surface.

A brief summary of the design of this roof according to the classical methods is presented in Appendix II. It is seen that the corrugated sheets and purlins have no interaction between them except to transfer loads.

According to the present approach, the flexural and torsional rigidities are calculated based on ¼-inch corrugated sheets and 5-inch beam. It is found that 3:

$$D_x = 7.65 \times 10^6 \text{ lb.in.}; D_{xy} = 2.97 \times 10^6 \text{ lb.in.}$$

$$D_y = 0.72 \times 10^6 \text{ lb.in.} \quad (21)$$

Half the roof in Figure 2(c) is represented by the equivalent grid system shown in Figure 3. Substitution of Equivalent 21 in Equivalents 16 to 20 yields:

for 48 × 48-inch panel

$E_dI_d = 14.5 \times 10^6$ lb.in.²

$E_cI_c = 180.0 \times 10^6$ lb.in²

$E_sI_s = 12.0 \times 10^6$ lb.in²

$G_sJ_s = 130.0 \times 10^6$ lb.in²

for 36 × 48-inch panel

$E_dI_d = 18.5 \times 10^6$ lb.in²

$E_cI_c = 130.0 \times 10^6$ lb.in²

$E_sI_s = 14.0 \times 10^6$ lb.in²

$G_cJ_c = 99.0 \times 10^6$ lb.in²

$G_sJ_s = 131.0 \times 10^6$ lb.in²

Using a general purpose computer program operating on the minicomputer VAX/VMS, it is found that the deflection at node six is 0.301 inches, whereas at the moment there is 28,700 inches per pound.

The discrepancies between the present solution and the classical one is −30 percent in terms of deflection and −15 percent for the moment. These ratios are not constants because the response of the roof varies from one position to another due to the two-way action introduced here. According to classical methods, the response of a purlin, for example, is the same whether it is supported on trusses only or on the top of a wall. Also, the previous ratios will be less once a finer grid is considered.

It is seen that the values obtained from classical theories overestimate the responses which lead to oversized members. In other words, the 5-inch I-beams which are not sufficient according to the design presented in Appendix II, can be chosen smaller by following the actual performance of the roof. Another aspect which now can be studied accurately is the dynamic performance of roofs. Unsymmetrical sections would not complicate any calculations in the present model whereas they have a great influence on the classic methods. The torsion of the purlins is considered here whereas it is ignored in the former approach. Last but not least, any roof now can be designed under different loading combinations to determine the worst condition which governs each of its elements.

The deformed shape of the roof under consideration is shown in Figure 4. Other drawings could have been produced with no difficulty on the mini-computer.

SUMMARY AND CONCLUSIONS

A new model to represent any roof for the purpose of using any of the available codes on mini- or micro-computers is presented. The new model simulates the actual behavior of roofs. No restrictions are imposed on axial, flexural and torsional rigidities of the system; boundary conditions; type of loadings; types of analysis and type of materials.

APPENDIX I-THE STIFFNESS COEFFICIENTS IN EQS. 6 TO 10

$$S_1 = (G_c j_c/k + E_s I_s + 4E_d I_d/r^3)/\lambda$$
$$S_2 = -G_c J_c/k\lambda$$
$$S_3 = 2E_s I_s/\lambda$$
$$S_4 = 2S_8/k = S_9 = 2S_{10} = 2_{S11} = $$
$$= \lambda^2 k S_{13}/3 = -2k\lambda S_{12}/3 =$$
$$= 2k\lambda S_{14}/3 = -2\lambda S_{15}/3 =$$
$$= 4kE_d I_d/r^3\lambda$$
$$S_5 = (4E_c I_c/k + G_s J_s + 4k^2 E_d I_d/r^3)/\lambda$$
$$S_6 = 2E_c I_c/k\lambda$$
$$S_7 = 12(E_c I_c/k^3 + E_s I_s + E_d I_d/r^3)/\lambda^3$$

in which

E = Young's modulus;
G = shear modulus;
I = moment of inertia;
J = torsional constant;
c, d, s = *subscripts denote chord, diagonal and side members, respectively*

APPENDIX II-CLASSICAL DESIGN OF THE ROOF IN FIGURE 2

Weight Estimate and Loadings

Roof covering = 3.0 psf of roof surface
Insulation board = 1½ psf
Purlins = 3.0 psf of roof surface

Snow load:
Snow load on a roof of 1/4 pitch =
20 psf of roof surface

Wind load:
Pressure on a roof of 1/4 pitch =
11.80 psf of roof surface

Design of Purlins

Two load combinations are considered: dead load plus full snow load, and dead load plus one-half snow load and full wind load.

- Dead load on a purlin = 420.0 lbs.
- Snow load on a purlin = 1120.0 lbs.
- Wind load on a purlin = 661.0 lbs.

The maximum loading in this case consists of dead load plus full snow load. When wind is considered, the snow load is reduced 50 percent and all working stresses are increased 33.33 percent at the same time. Hence the wind load is not large enough to influence the section of the purlin.

- Moment about major axis at center of purlin = 32,890.0 in. lb.
- Moment about minor axis at center of purlin = 4,160.0 in. lb.
- Try a 5-inch, 10-lb. standard I-beam
- Maximum stress = 11,870.0 psi.
- Maximum deflection = 0.391 inch.

The 5-inch I-beam satisfies the requirements of AISC. However, if a depth 1-30 of the span should be thought desirable to avoid all possibility of slight vibration during a high gusty wind, it will be found that the 8-inch × 4-inch joist section at 10 pounds per foot is safe according to the AISC Manual.

REFERENCES

[1] Lekhnitskii, S.G., Anistropic Plates, Gordon & Breach, Science Publishers, Inc. New York, 1968.

[2] Timoshenko, S. and Woinowsky-Kriegez, S., Theory of Plates and Shells, McGraw-Hill Book Company, New York, 1968.

[3] Troitsky, M.S., Stiffened Plates, Bending, Stability and Vibrations, Elsevier Scientific Publishing Co., New York, 1976.

NOTATIONS

The following symbols are used in this paper:

D_x, D_y = flexural rigidity of a roof element;
D_{xy} = torsional rigidity of a roof element;
D_x, D_y = flexural rigidity of a roof element;
E = Young's modulus;
G = shear modulus;
GJ = torsional rigidity of a grid element;
H = torque intensity;
I = moment of inertia;
M = moment intensity;
S_{ij} = an element in stiffness matrix;
X, Y = coordinate axes;
λ, $k\lambda$ = element side dimensions.

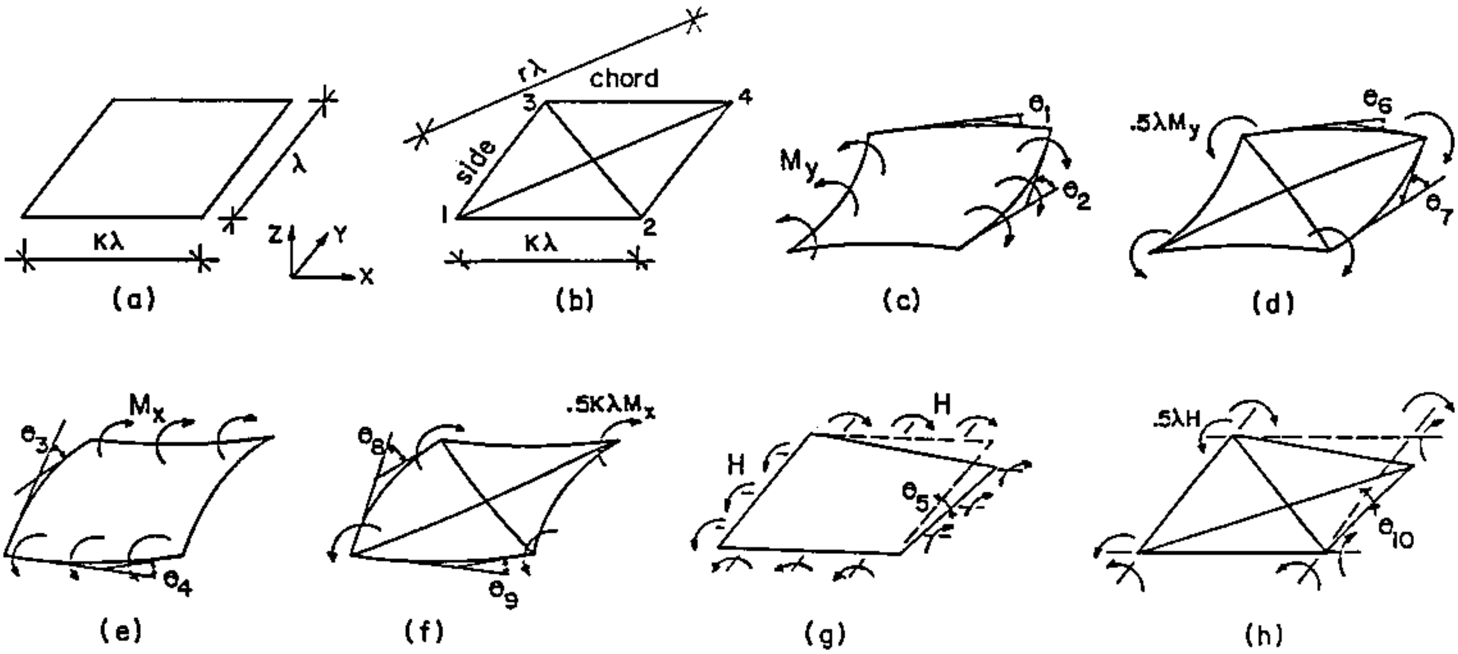

Figure 1   A roof element and its equivalent model

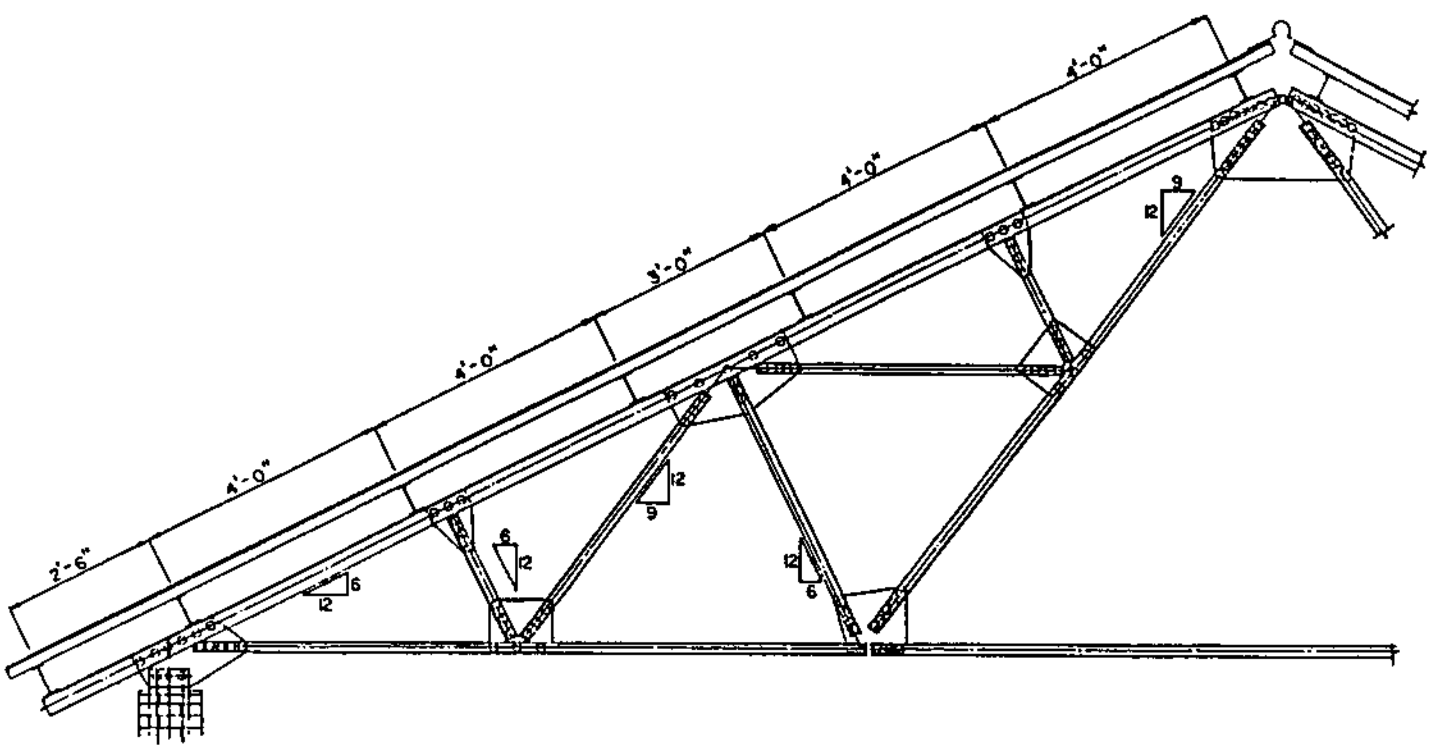

Figure 2(a)   The roof system used for numerical results

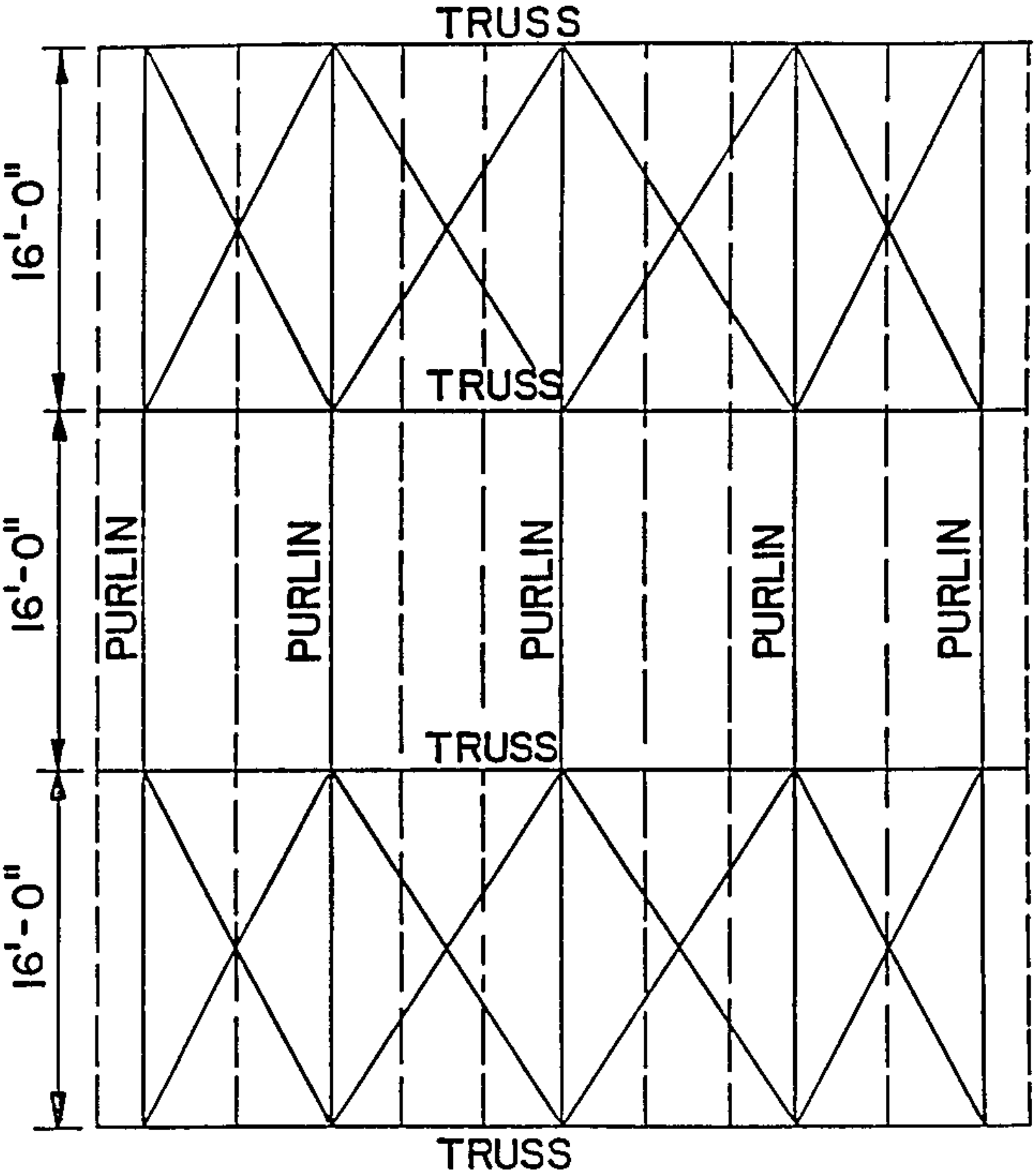

Figure 2(b) A plane view of the roof system in Figure 2(a)

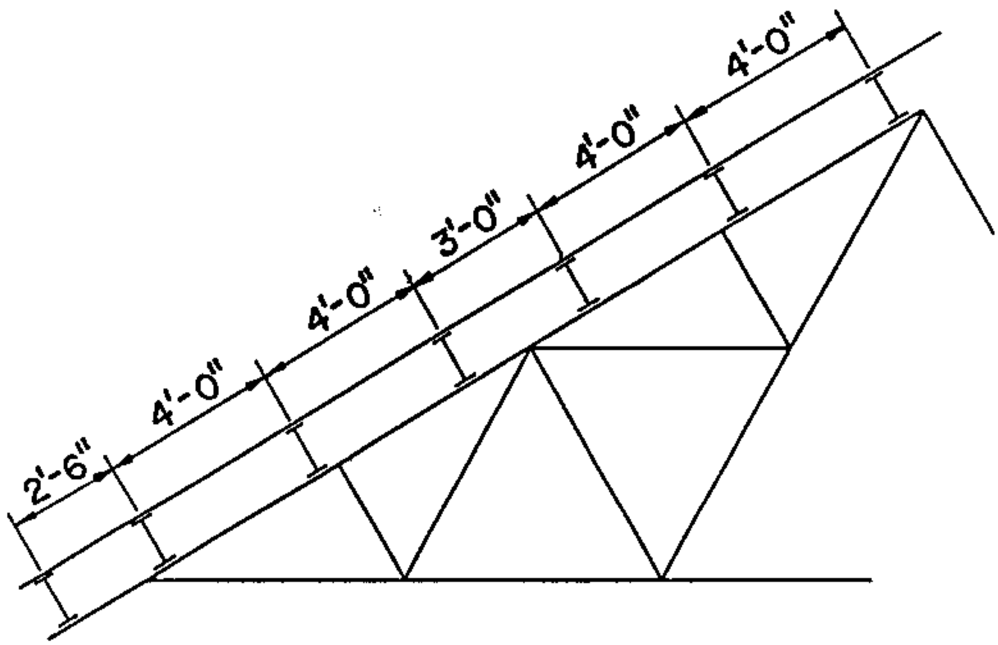

Figure 2(c) A schematic presentation of the roof system in Figure 2(a)

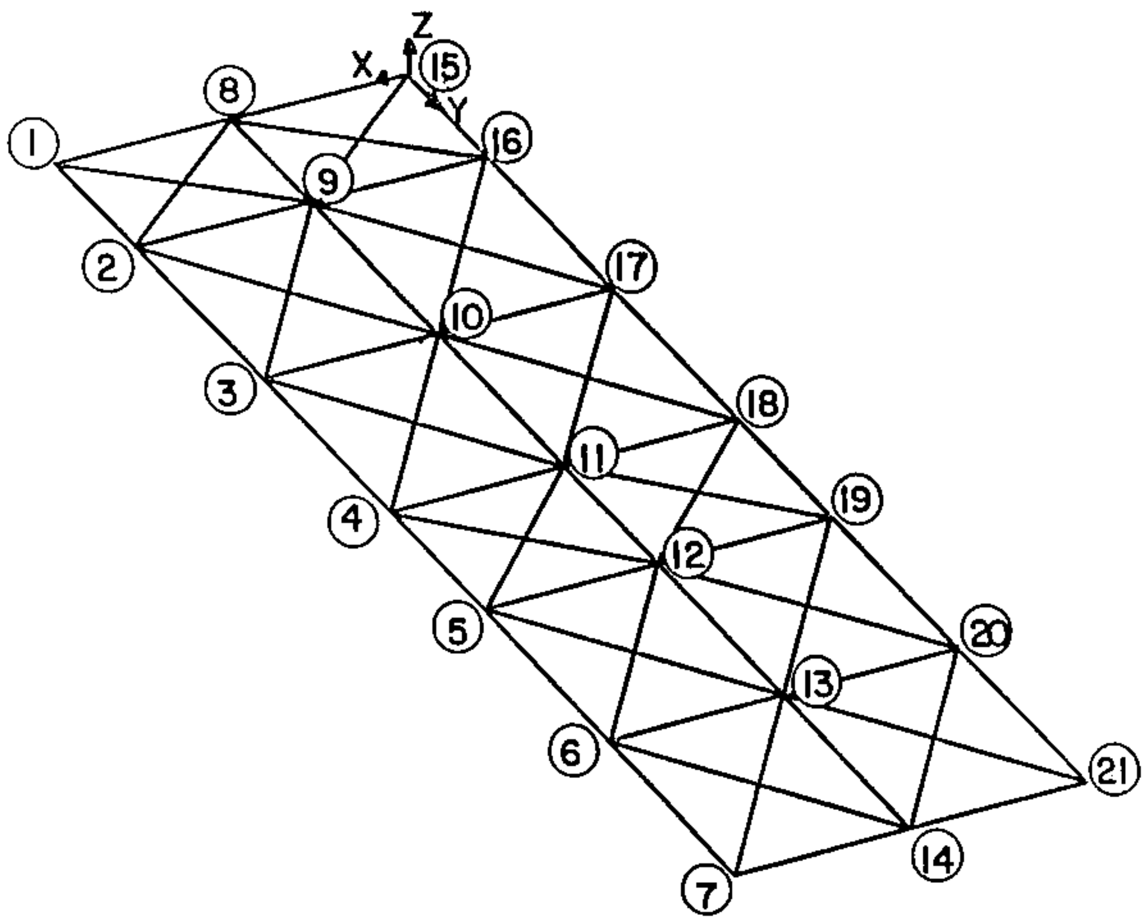

Figure 3 Equivalent system for the roof in Figure 2(a)

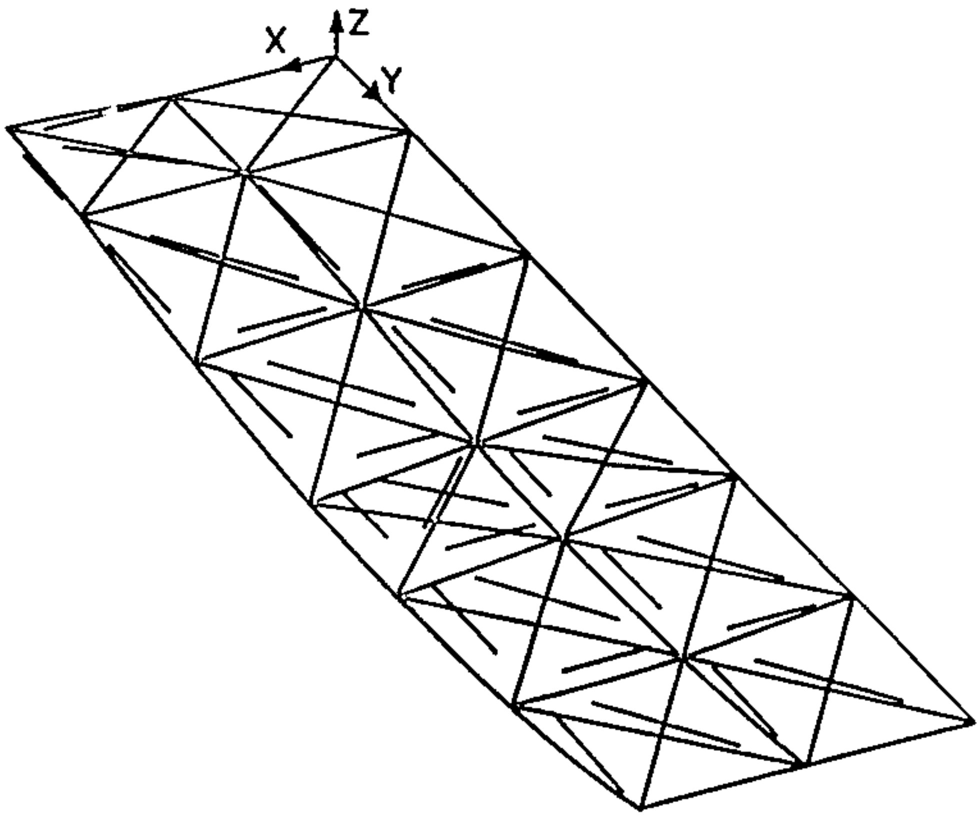

Figure 4 *The deformed shape of the roof in Figure 2(a)*